Quantum Sensing

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CSRC

BNU, 2022/05/13

Quantum Sensing

1. What ?

2. Why ?

3. How ?

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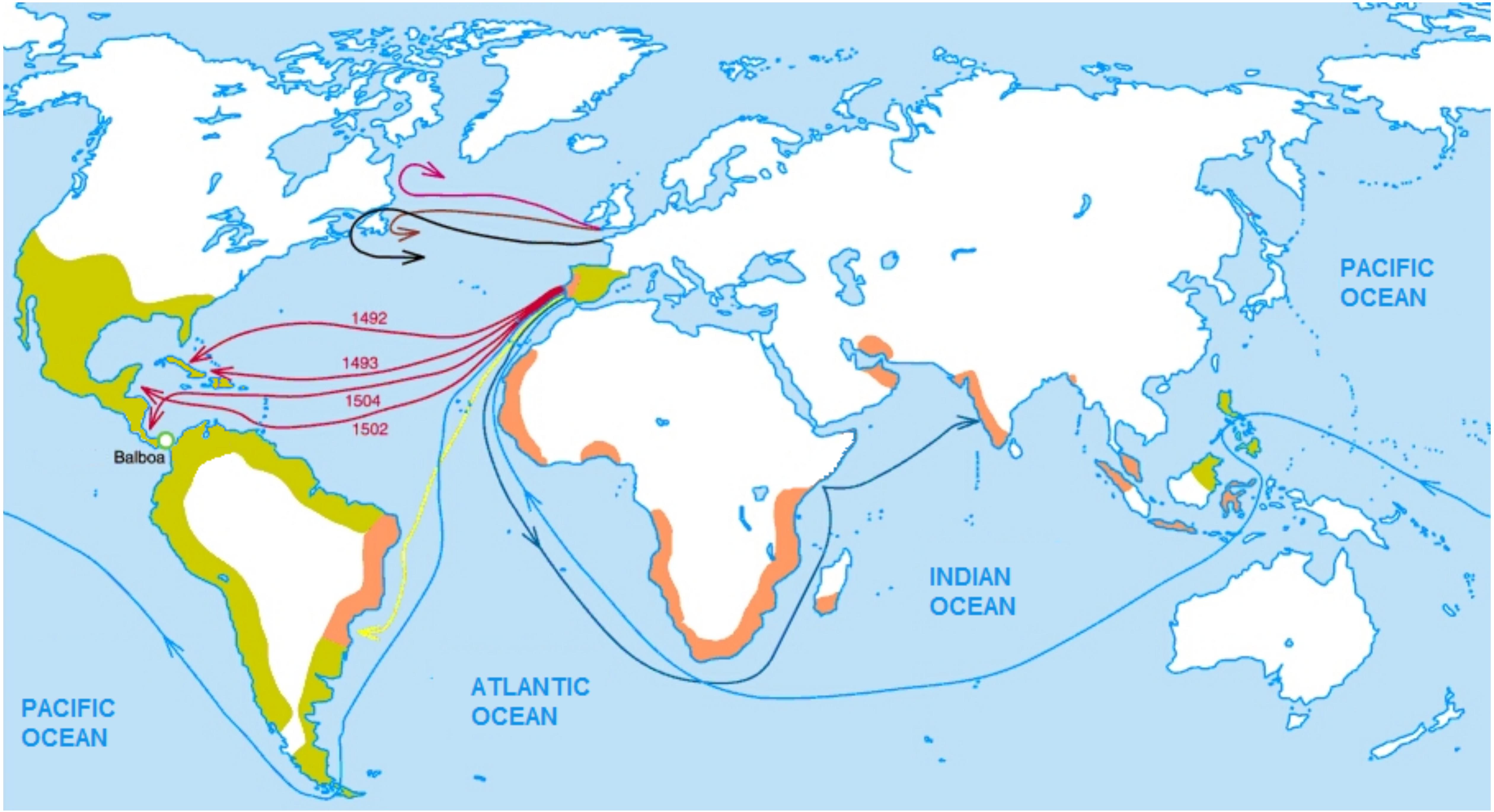
What is ~~Quantum~~ Sensing?

measuring of a physical quantity using a device (sensor)

sound mirror before the invention of **Radar**

“human” sensor

The Age of Discovery



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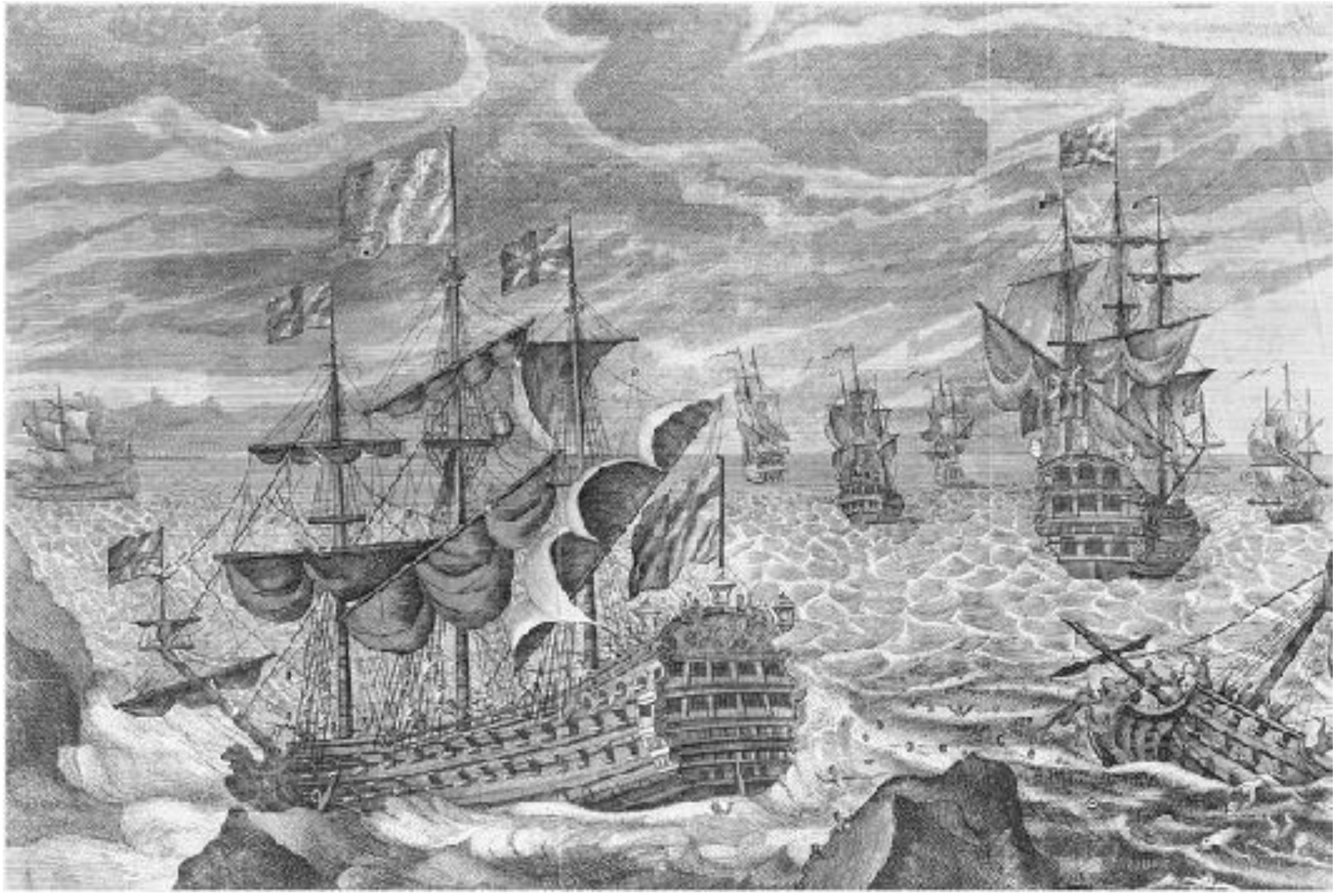
Navigation !

Navigation and Timekeeping



The Loss of British Fleet (1707)

Four warships of a Royal Navy fleet were lost at *the Isles of Scilly* on **22 October 1707**. About 2,000 sailors died in this disaster.

The disaster has been attributed to a combination of factors, including the navigators' inability to accurately **calculate the positions**, errors in the available **charts and pilot books**, and **inadequate compasses**.

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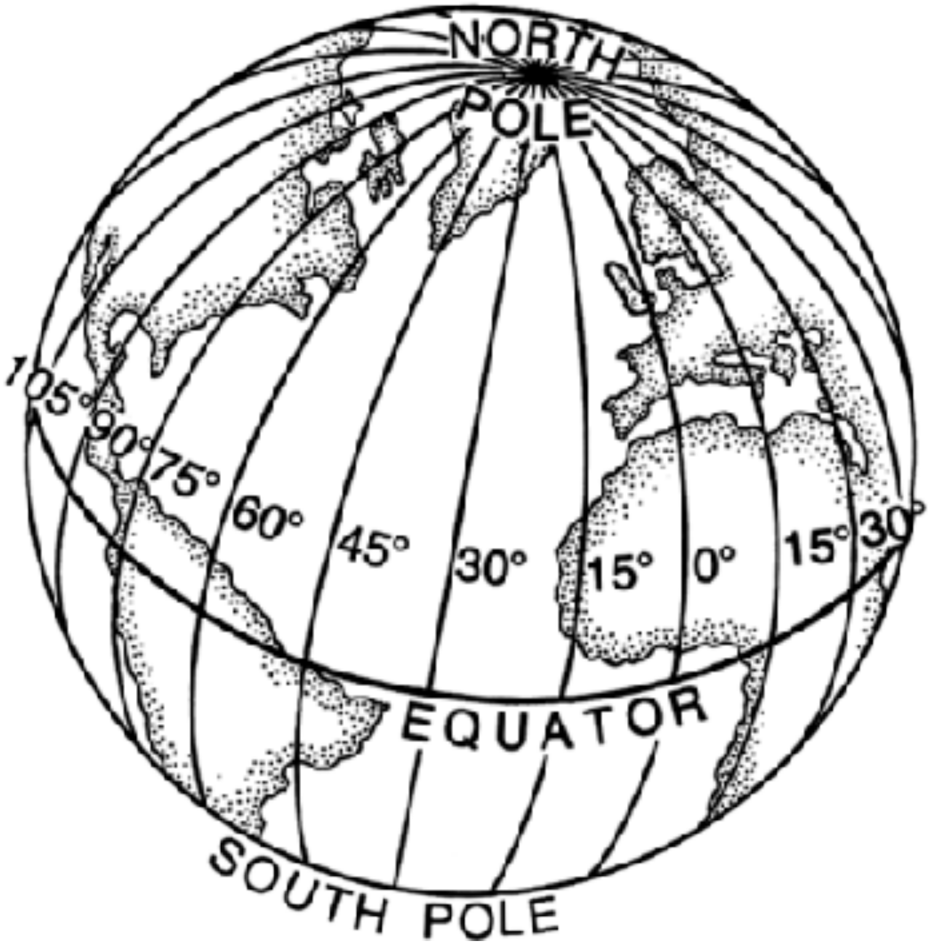
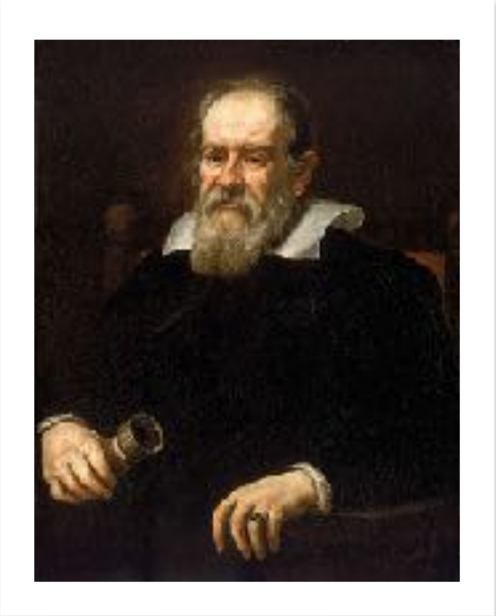
Navigation and Timekeeping



The Longitude Problem


Galileo


Huygens


Newton


Halley

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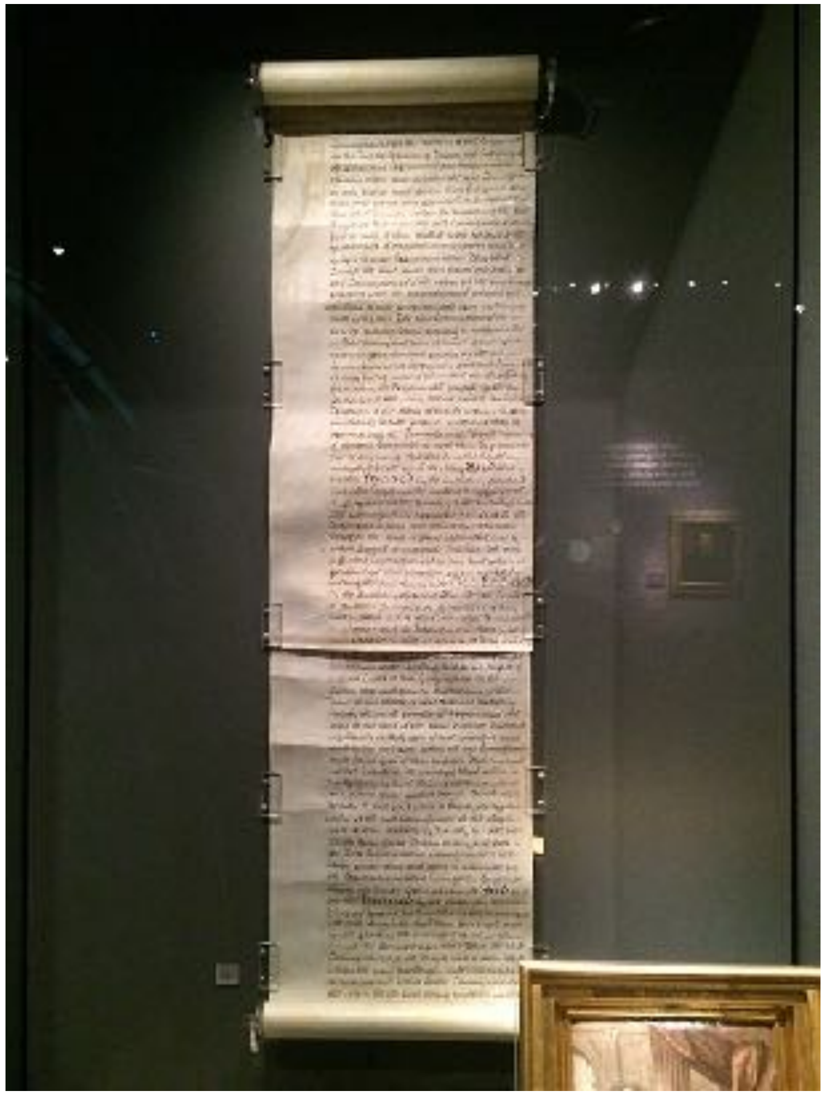
The Longitude Act



The Longitude Act of 1714

- **1st prize** (£20,000)

1/2 degree of longitude (30 nautical miles)

- **2nd prize** (£15,000)

2/3 degree of longitude (45 nautical miles)

- **3rd prize** (£10,000)

1 degree of longitude (60 nautical miles)

1 nautical miles ≈ 1.8 kilometers
£10,000 in 1714 ≈ *£1.5 million* in 2020

What is ~~Quantum~~ Sensing?

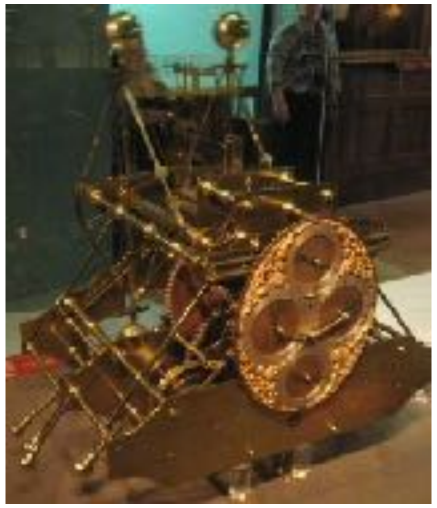
John Harrison

1693-1776

Inventor of the

Marine Chronometer

（航海钟）



H1

H2

H3

H4

H5

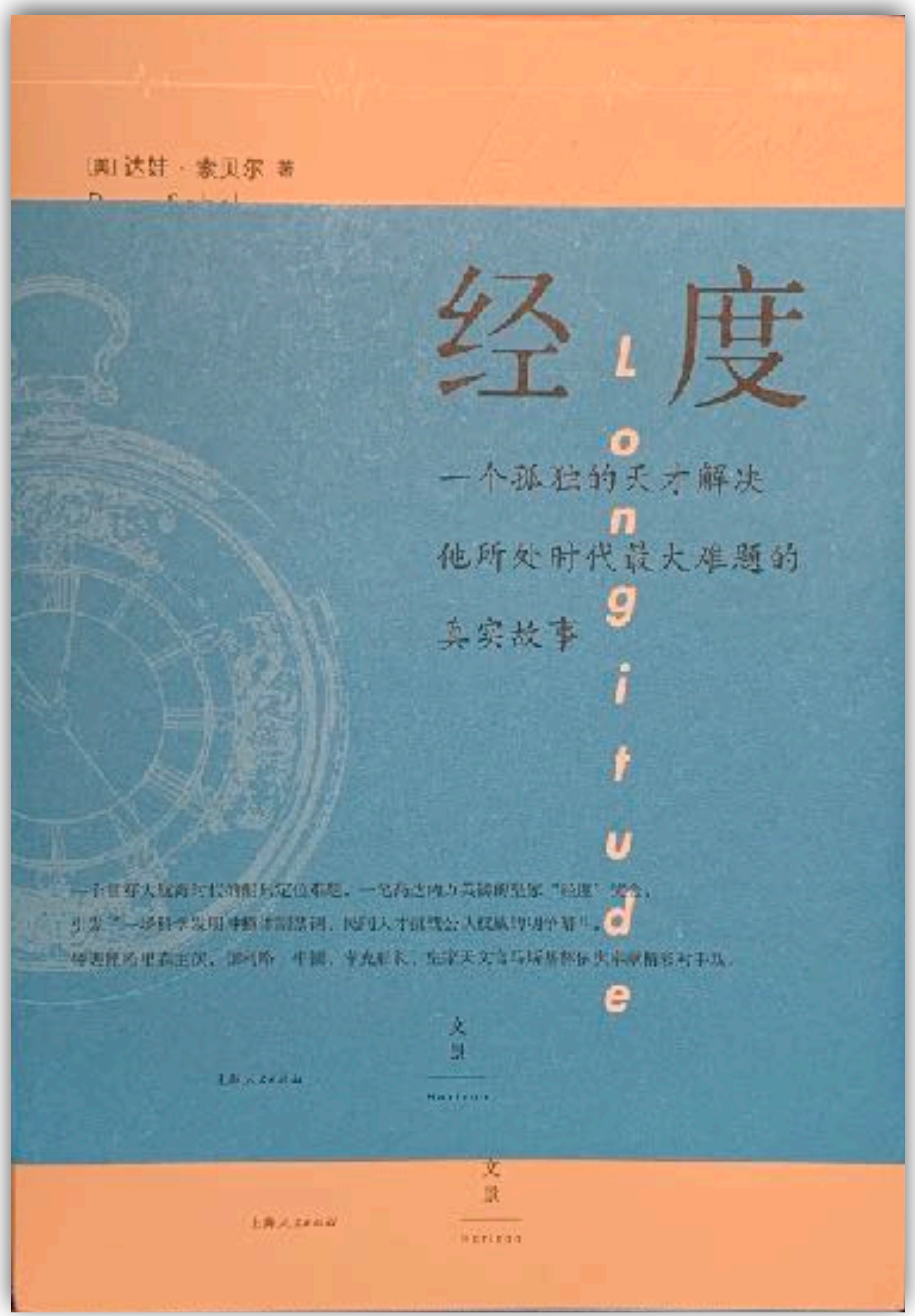
1735 1739 1758 1759 1772

What is ~~Quantum~~ Sensing?

accuracy

$0.2\ \mathrm{s/day} = 2.3 \times 10^{-6}$

Shortly before he died, Harrison received nearly the full £20,000.



[美] 达娃·索贝尔 著

经度

Longitude

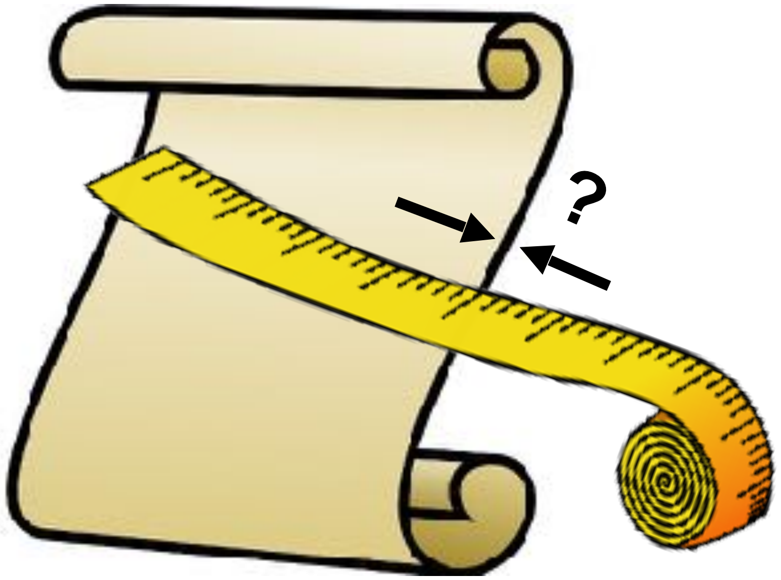
一个孤独的天才解决他所处时代最大难题的真实故事

What is ~~Quantum~~ Sensing?

physical quantities to be measured

- time, frequency, …
- position, velocity, acceleration, rotation, …
- electric field, magnetic field, electromagnetic wave, …
- temperature, strain, pressure, …

Quiz: how to measure the thickness of a piece of paper?



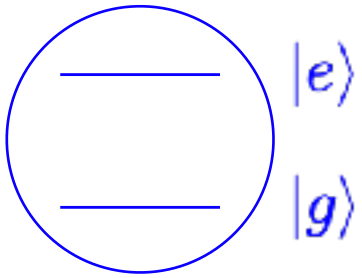
what kind of **equipment** do you need?

what kind of **"resource"** do you need?

What is Quantum Sensing?

Quantum sensing is typically used to describe one of the following:

I. Use of a **quantum object** to measure a physical quantity (classical or quantum);



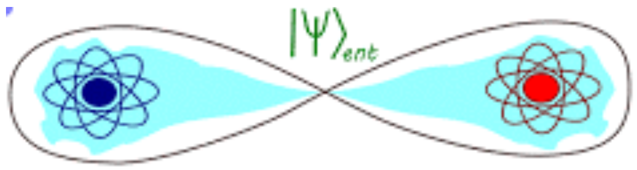
discrete levels

II. Use of **quantum coherence** (i.e., wavelike spatial or temporal superposition states) to measure a physical quantity;

$\alpha|e\rangle + \beta|g\rangle$

quantum superposition

III. Use of **quantum entanglement** to improve the sensitivity or precision of a measurement, *beyond what is possible classically*.



quantum entanglement

only *type-III* is strictly "quantum"

Degen, C. L., Reinhard, F. & Cappellaro, P. Quantum sensing. *Rev. Mod. Phys*. **89**, 035002 (2017)

What is Quantum Sensing?

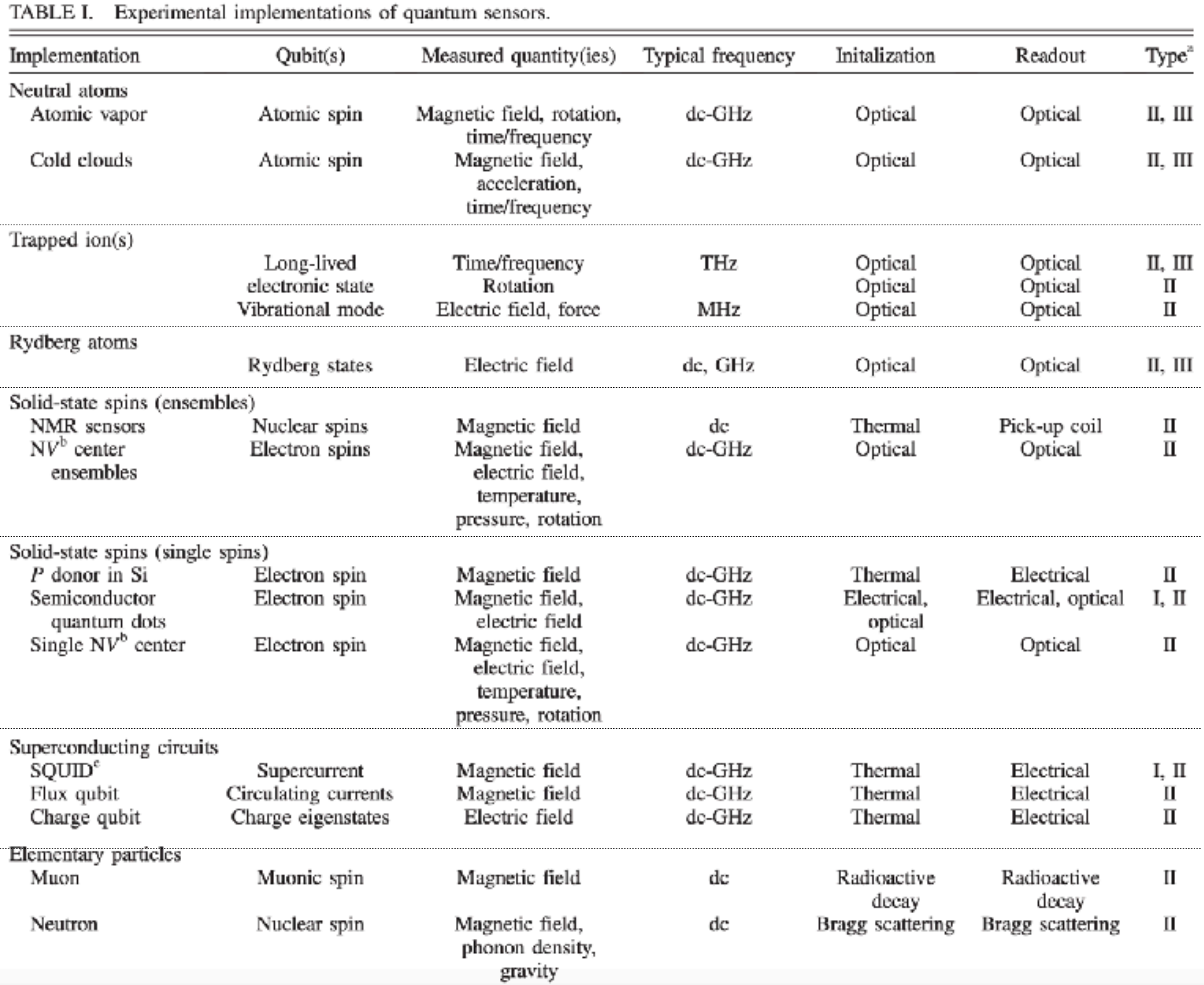
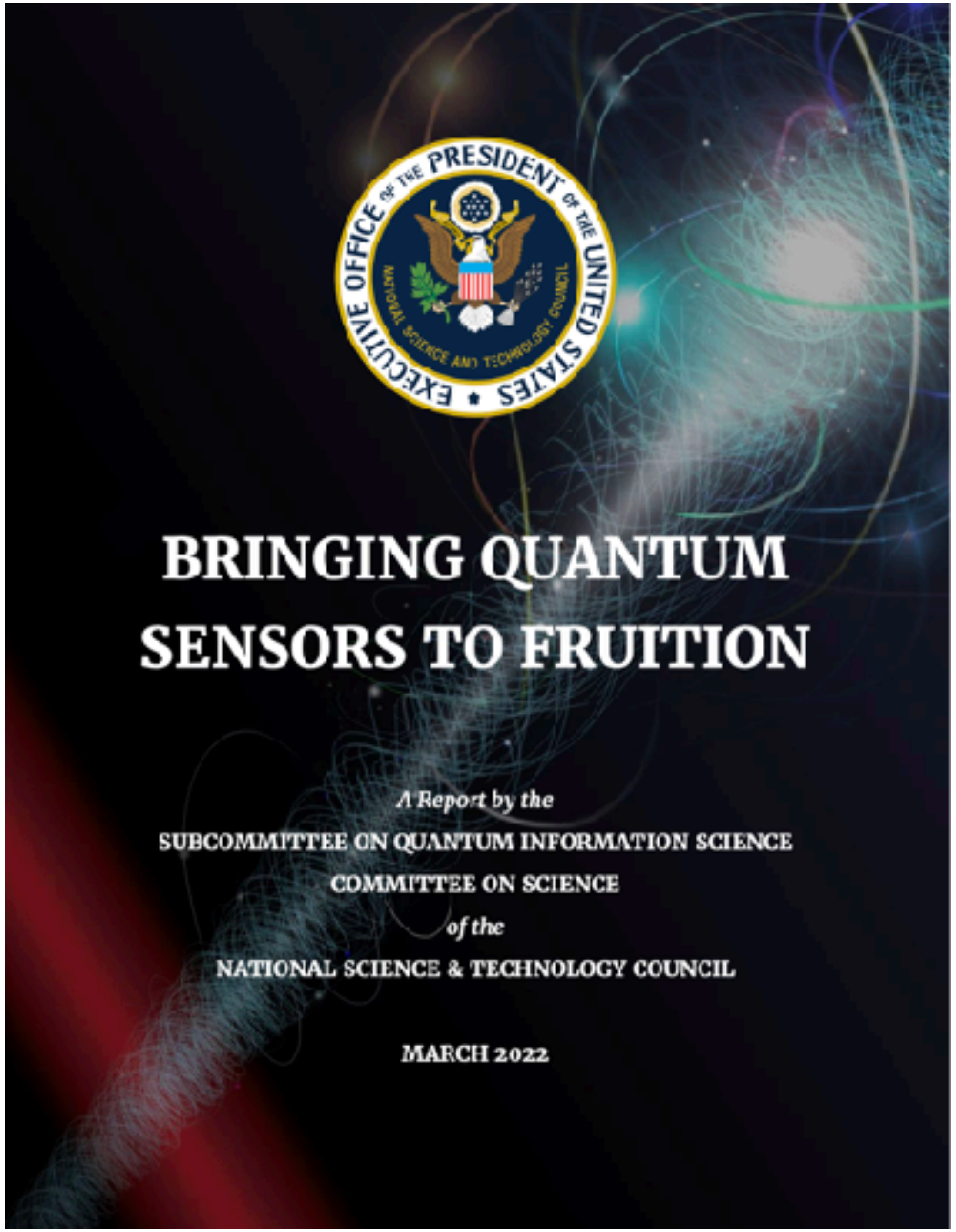


TABLE I. Experimental implementations of quantum sensors.

Implementation	Qubit(s)	Measured quantity(ies)	Typical frequency	Initalization	Readout	Type[a]
Neutral atoms						
Atomic vapor	Atomic spin	Magnetic field, rotation, time/frequency	dc-GHz	Optical	Optical	II, III
Cold clouds	Atomic spin	Magnetic field, acceleration, time/frequency	dc-GHz	Optical	Optical	II, III
Trapped ion(s)						
	Long-lived electronic state	Time/frequency	THz	Optical	Optical	II, III
		Rotation		Optical	Optical	II
	Vibrational mode	Electric field, force	MHz	Optical	Optical	II
Rydberg atoms						
	Rydberg states	Electric field	dc, GHz	Optical	Optical	II, III
Solid-state spins (ensembles)						
NMR sensors	Nuclear spins	Magnetic field	dc	Thermal	Pick-up coil	II
NV[b] center ensembles	Electron spins	Magnetic field, electric field, temperature, pressure, rotation	dc-GHz	Optical	Optical	II
Solid-state spins (single spins)						
P donor in Si	Electron spin	Magnetic field	dc-GHz	Thermal	Electrical	II
Semiconductor quantum dots	Electron spin	Magnetic field, electric field	dc-GHz	Electrical, optical	Electrical, optical	I, II
Single NV[b] center	Electron spin	Magnetic field, electric field, temperature, pressure, rotation	dc-GHz	Optical	Optical	II
Superconducting circuits						
SQUID[c]	Supercurrent	Magnetic field	dc-GHz	Thermal	Electrical	I, II
Flux qubit	Circulating currents	Magnetic field	dc-GHz	Thermal	Electrical	II
Charge qubit	Charge eigenstates	Electric field	dc-GHz	Thermal	Electrical	II
Elementary particles						
Muon	Muonic spin	Magnetic field	dc	Radioactive decay	Radioactive decay	II
Neutron	Nuclear spin	Magnetic field, phonon density, gravity	dc	Bragg scattering	Bragg scattering	II

Examples of Quantum Sensing



EXECUTIVE OFFICE OF THE PRESIDENT OF THE UNITED STATES

BRINGING QUANTUM SENSORS TO FRUITION

A Report by the

SUBCOMMITTEE ON QUANTUM INFORMATION SCIENCE

COMMITTEE ON SCIENCE

of the

NATIONAL SCIENCE & TECHNOLOGY COUNCIL

MARCH 2022

- atomic clock
- atom interferometer
- optical magnetometer
- Rydberg atoms
- systems using quantum resources

Atomic Clock

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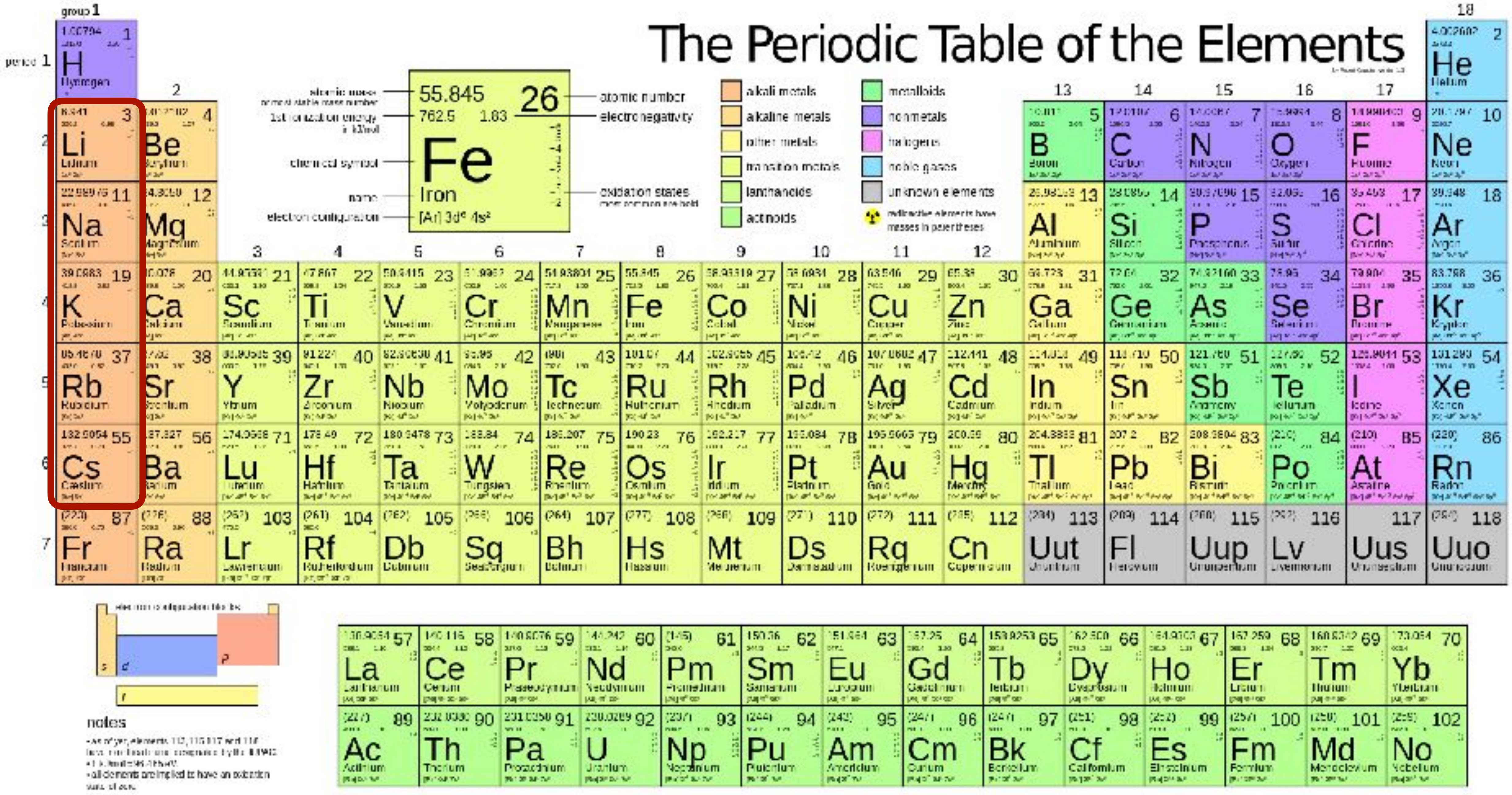
Examples of Quantum Sensing: Atomic Clock

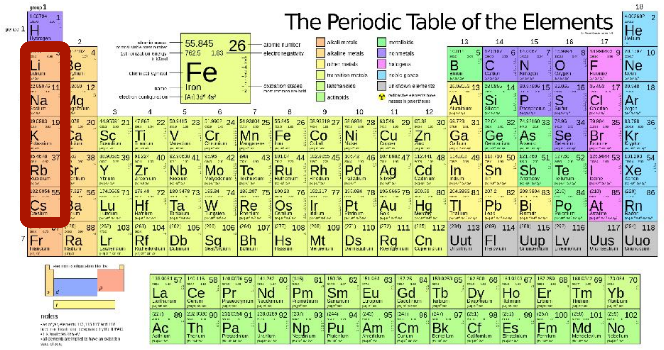


The Periodic Table of the Elements

group 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H Hydrogen																	2 He Helium
3 Li Lithium	4 Be Beryllium											5 B Boron	6 C Carbon	7 N Nitrogen	8 O Oxygen	9 F Fluorine	10 Ne Neon
11 Na Sodium	12 Mg Magnesium											13 Al Aluminium	14 Si Silicon	15 P Phosphorus	16 S Sulfur	17 Cl Chlorine	18 Ar Argon
19 K Potassium	20 Ca Calcium	21 Sc Scandium	22 Ti Titanium	23 V Vanadium	24 Cr Chromium	25 Mn Manganese	26 Fe Iron	27 Co Cobalt	28 Ni Nickel	29 Cu Copper	30 Zn Zinc	31 Ga Gallium	32 Ge Germanium	33 As Arsenic	34 Se Selenium	35 Br Bromine	36 Kr Krypton
37 Rb Rubidium	38 Sr Strontium	39 Y Yttrium	40 Zr Zirconium	41 Nb Niobium	42 Mo Molybdenum	43 Tc Technetium	44 Ru Ruthenium	45 Rh Rhodium	46 Pd Palladium	47 Ag Silver	48 Cd Cadmium	49 In Indium	50 Sn Tin	51 Sb Antimony	52 Te Tellurium	53 I Iodine	54 Xe Xenon
55 Cs Caesium	56 Ba Barium	71 Lu Lutetium	72 Hf Hafnium	73 Ta Tantalum	74 W Tungsten	75 Re Rhenium	76 Os Osmium	77 Ir Iridium	78 Pt Platinum	79 Au Gold	80 Hg Mercury	81 Tl Thallium	82 Pb Lead	83 Bi Bismuth	84 Po Polonium	85 At Astatine	86 Rn Radon
87 Fr Francium	88 Ra Radium	103 Lr Lawrencium	104 Rf Rutherfordium	105 Db Dubnium	106 Sg Seaborgium	107 Bh Bohrium	108 Hs Hassium	109 Mt Meitnerium	110 Ds Darmstadtium	111 Rg Roentgenium	112 Cn Copernicium	113 Uut Ununtrium	114 Fl Flerovium	115 Uup Ununpentium	116 Lv Livermorium	117 Uus Ununseptium	118 Uuo Ununoctium

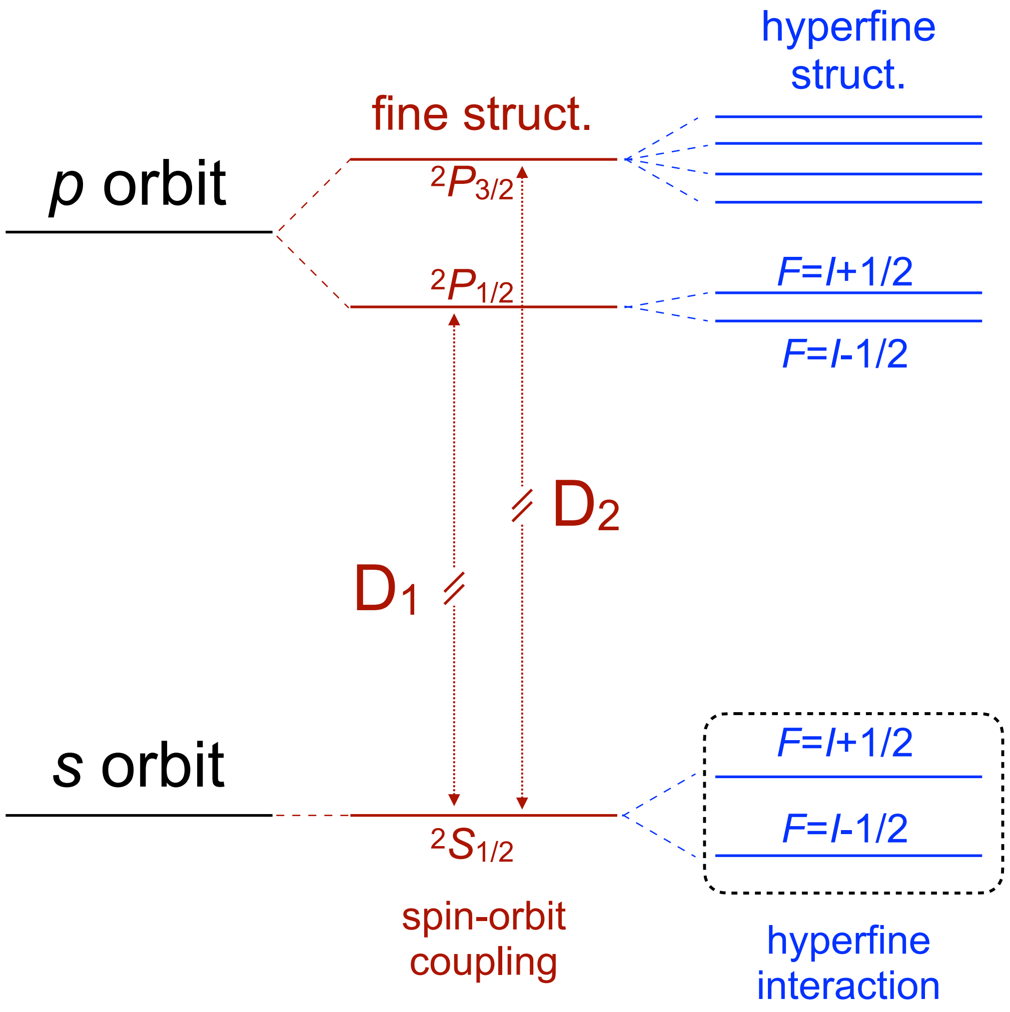
57 La Lanthanum	58 Ce Cerium	59 Pr Praseodymium	60 Nd Neodymium	61 Pm Promethium	62 Sm Samarium	63 Eu Europium	64 Gd Gadolinium	65 Tb Terbium	66 Dy Dysprosium	67 Ho Holmium	68 Er Erbium	69 Tm Thulium	70 Yb Ytterbium
89 Ac Actinium	90 Th Thorium	91 Pa Protactinium	92 U Uranium	93 Np Neptunium	94 Pu Plutonium	95 Am Americium	96 Cm Curium	97 Bk Berkelium	98 Cf Californium	99 Es Einsteinium	100 Fm Fermium	101 Md Mendelevium	102 No Nobelium

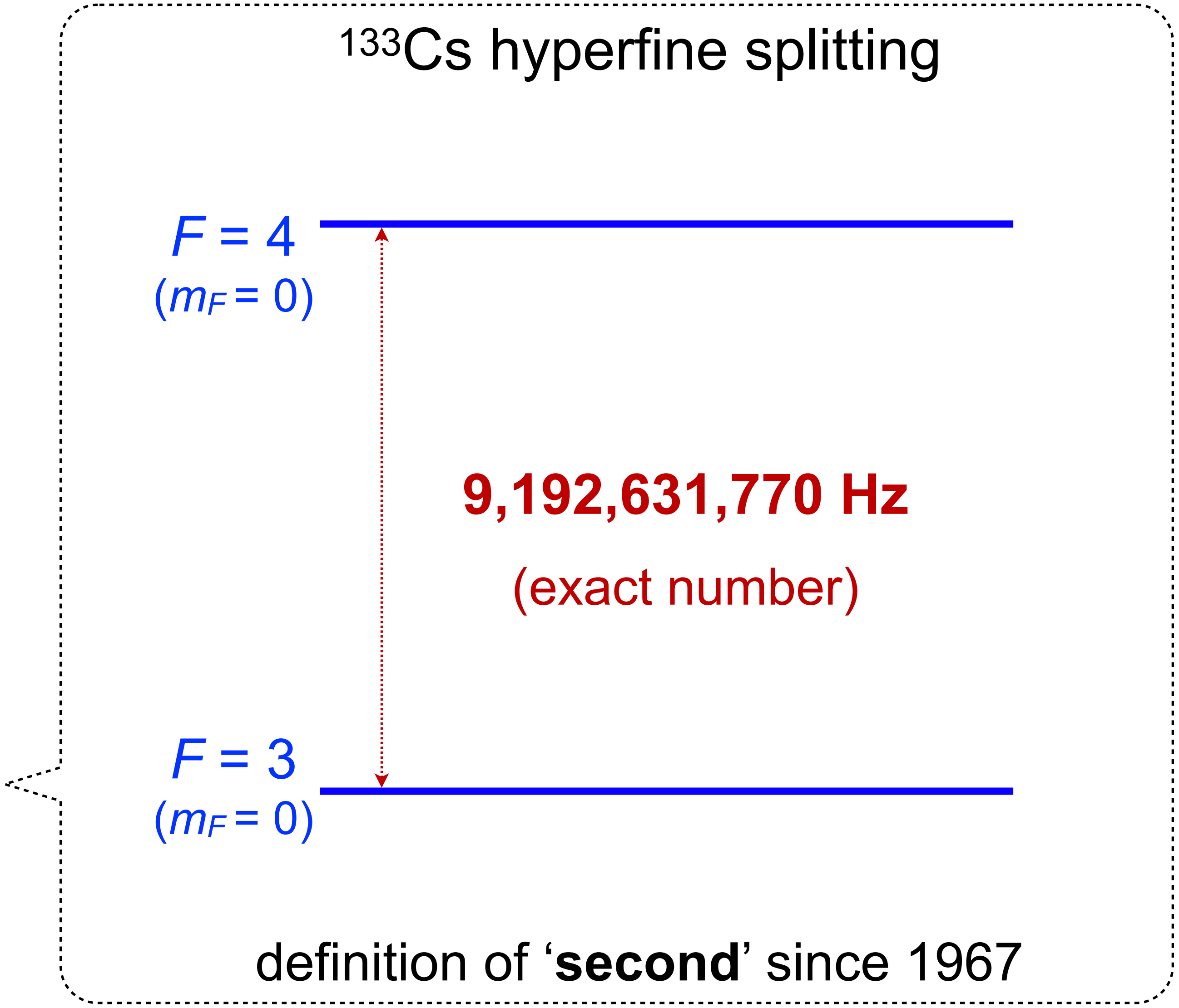
Alkali Metal



electron configuration $[X]\ ns^1$

X=He, Ne, Ar… $n = 2, 3, 4 \ldots$





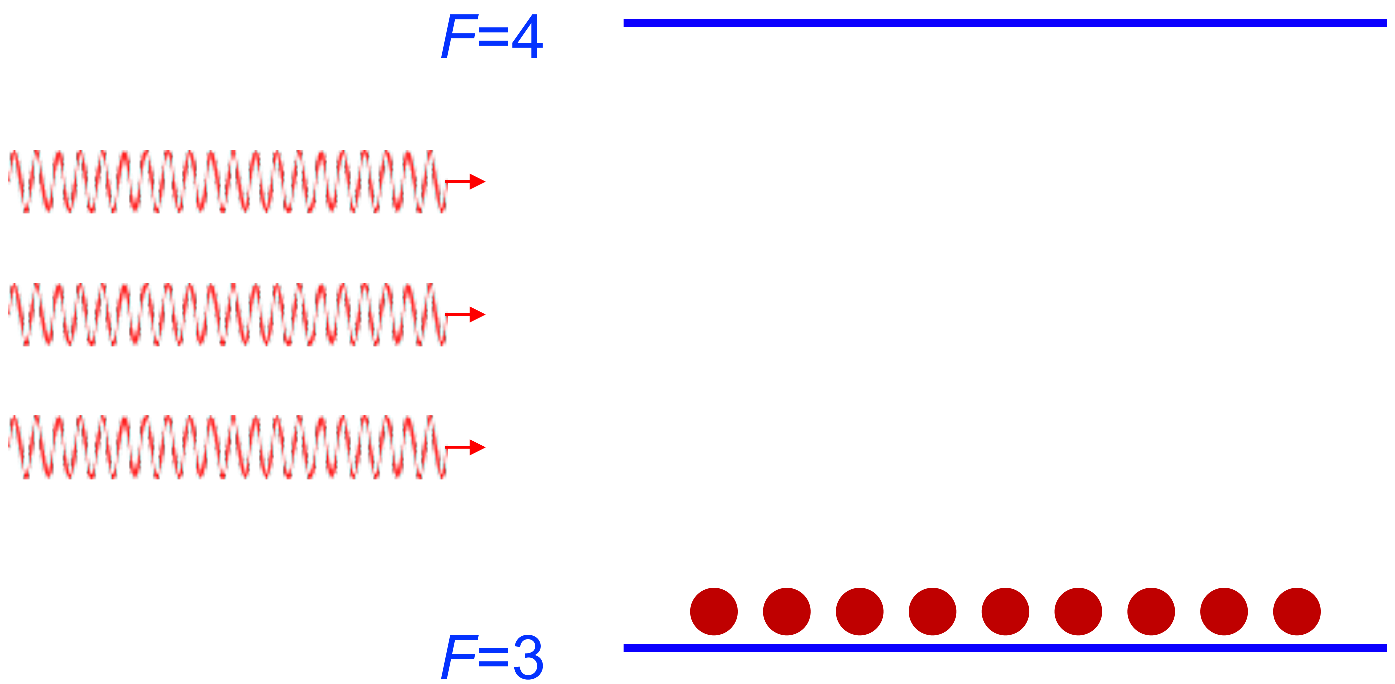
^{133}Cs hyperfine splitting

9,192,631,770 Hz

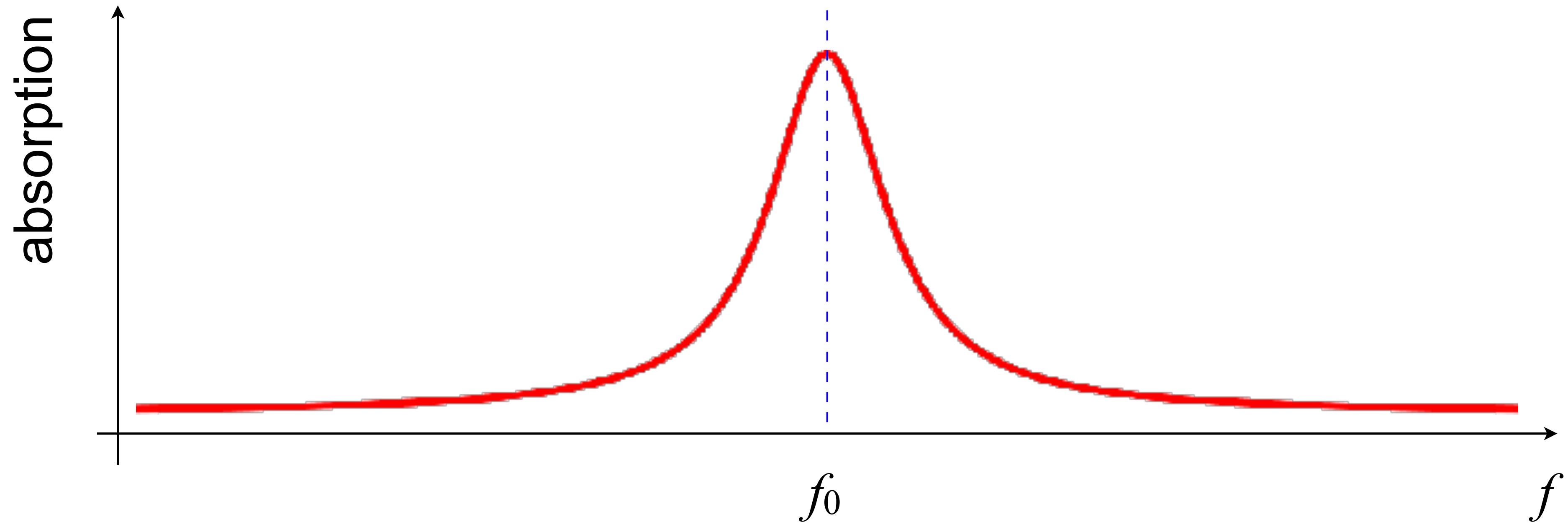
(exact number)

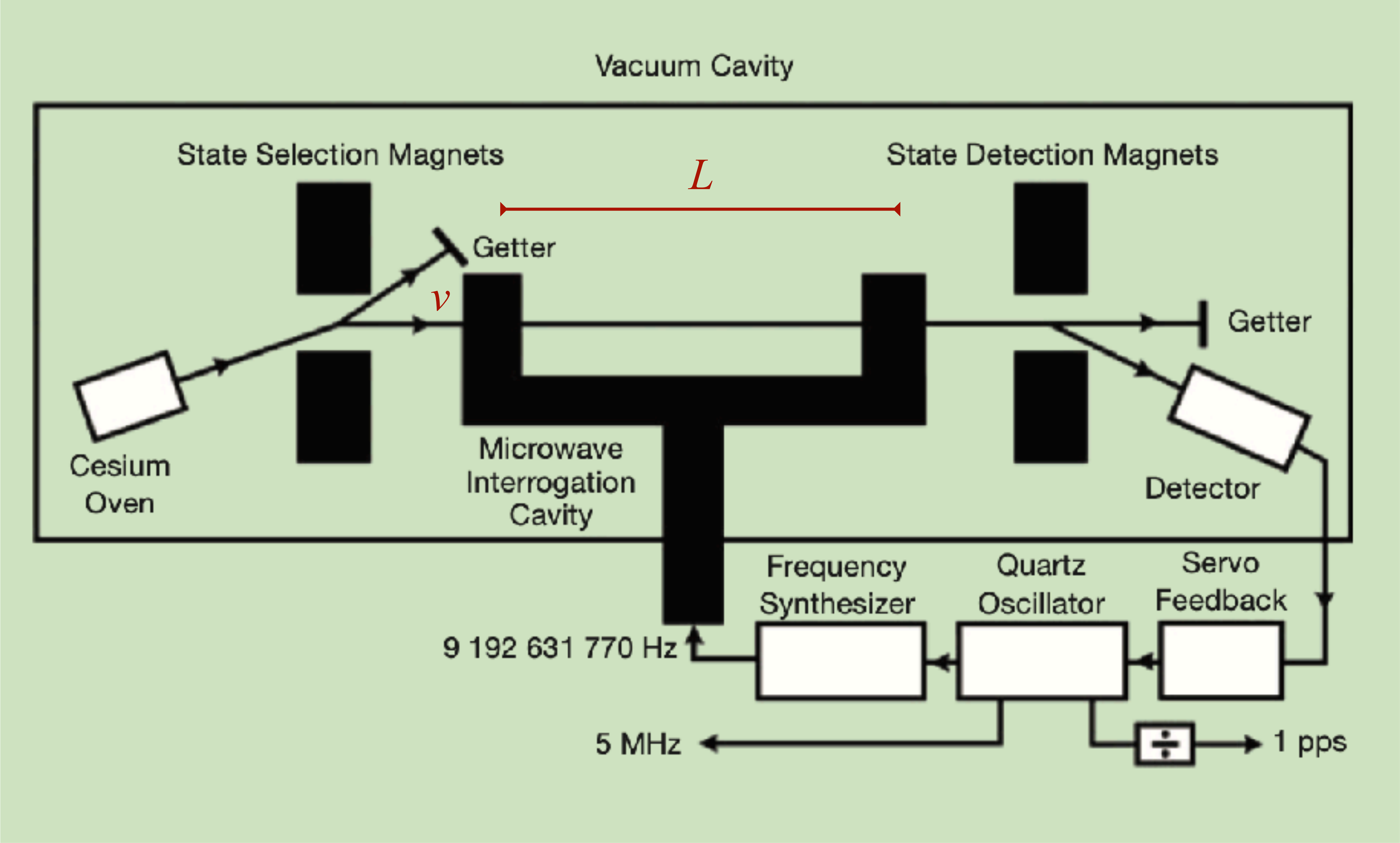
definition of ‘**second**’ since 1967

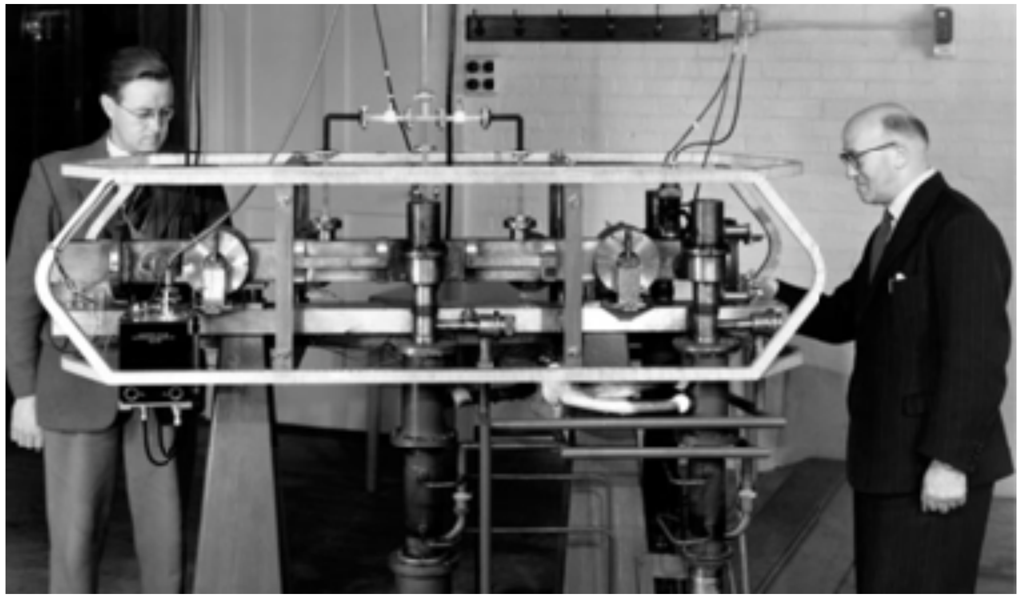
f-tunable
microwave
source



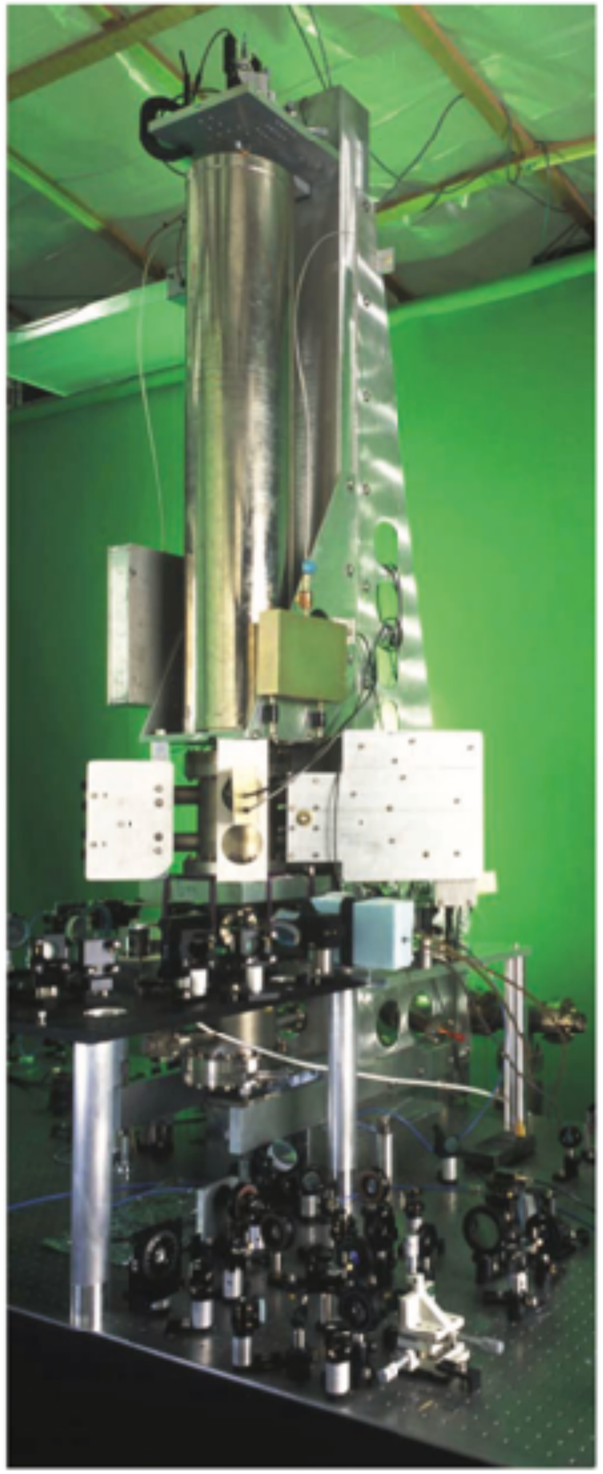
absorbed
microwave




Vacuum Cavity
State Selection Magnets
L
State Detection Magnets
Getter
v
Getter
Cesium Oven
Microwave Interrogation Cavity
Detector
Frequency Synthesizer
Quartz Oscillator
Servo Feedback
9 192 631 770 Hz
5 MHz
÷
1 pps



Louis Essen (right) and Jack Parry (left) standing next to the world's first ^{133}Cs atomic clock



Cs fountain clock NIST-F1

NIST physicist John Kitching and world's smallest chip-scale atomic clock (CSAC)

Atom Interferometer

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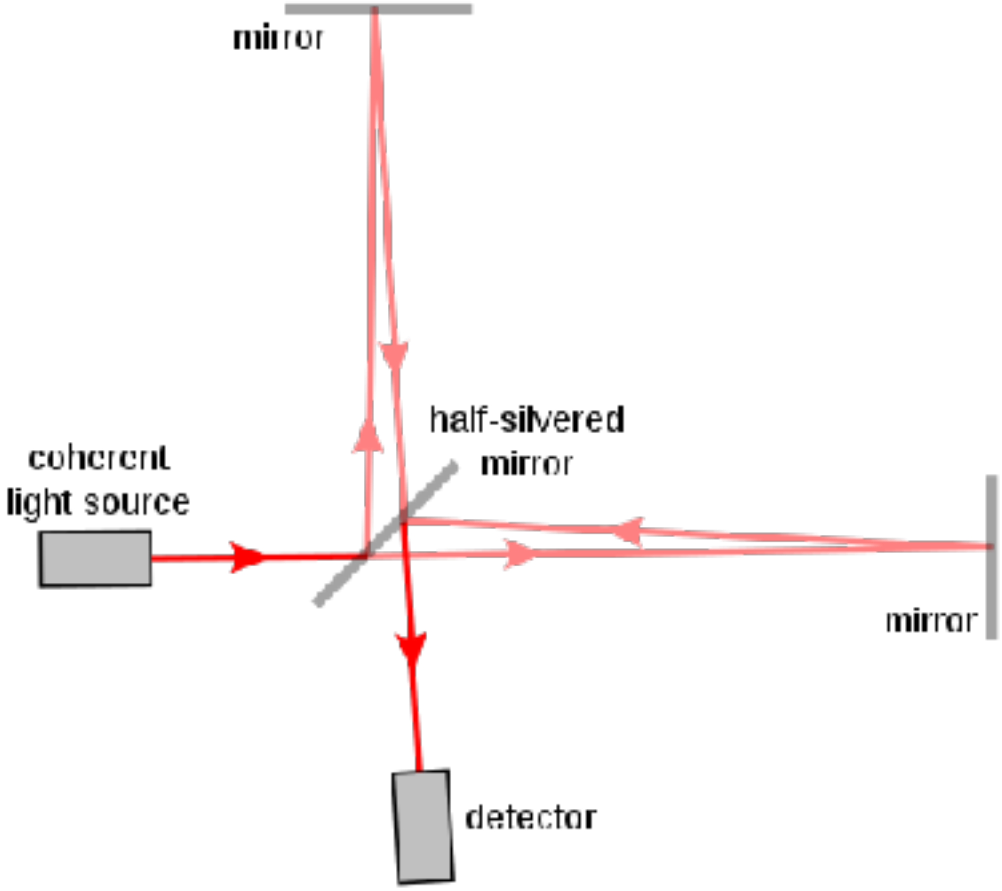
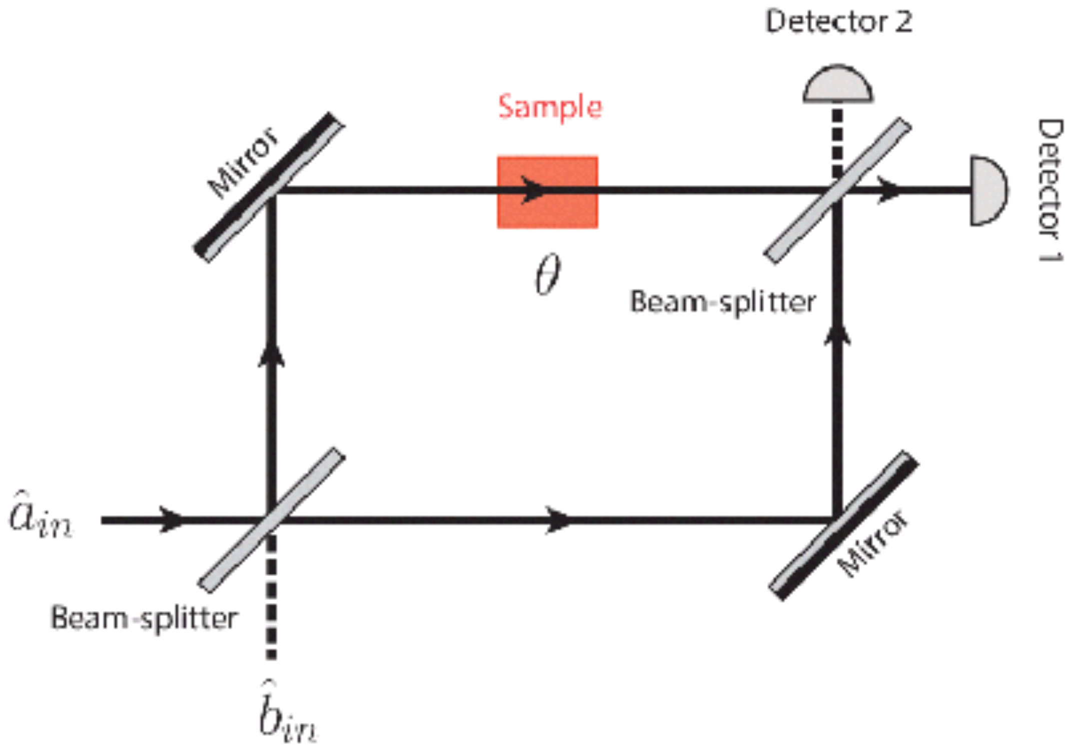
Examples of Quantum Sensing: Atom Interferometer

Optical Interferometer

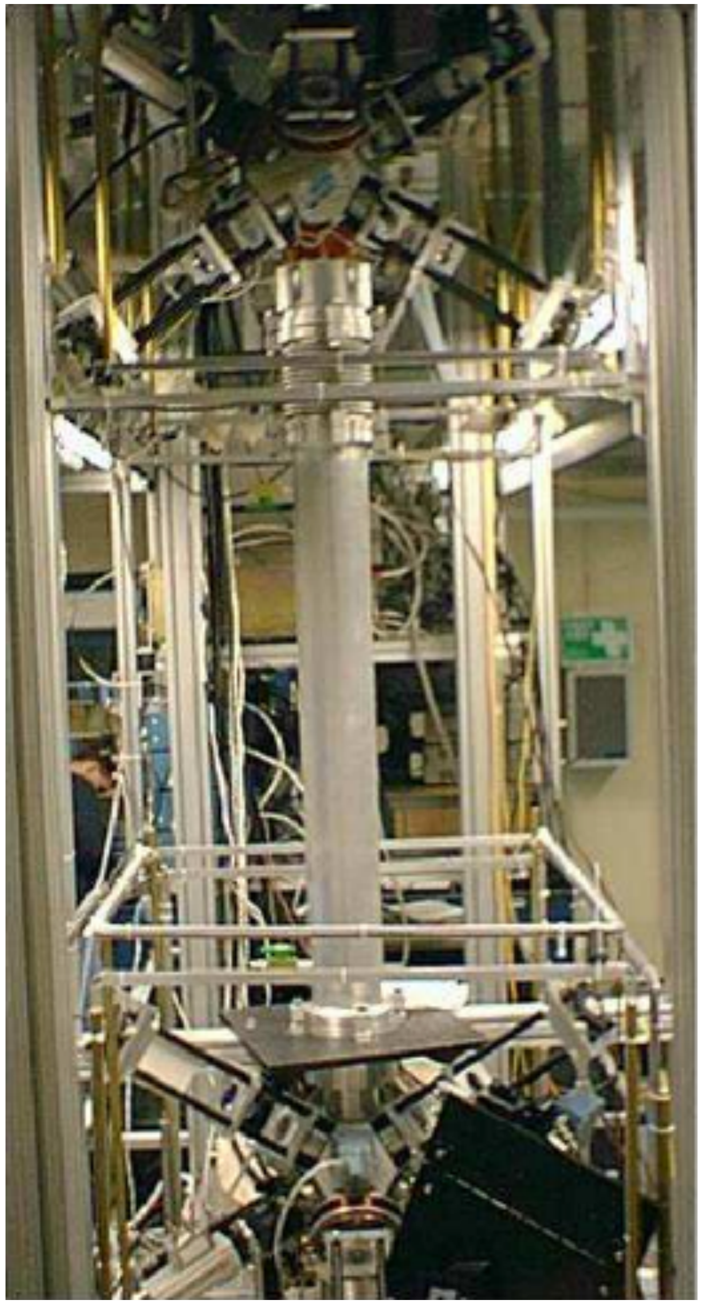


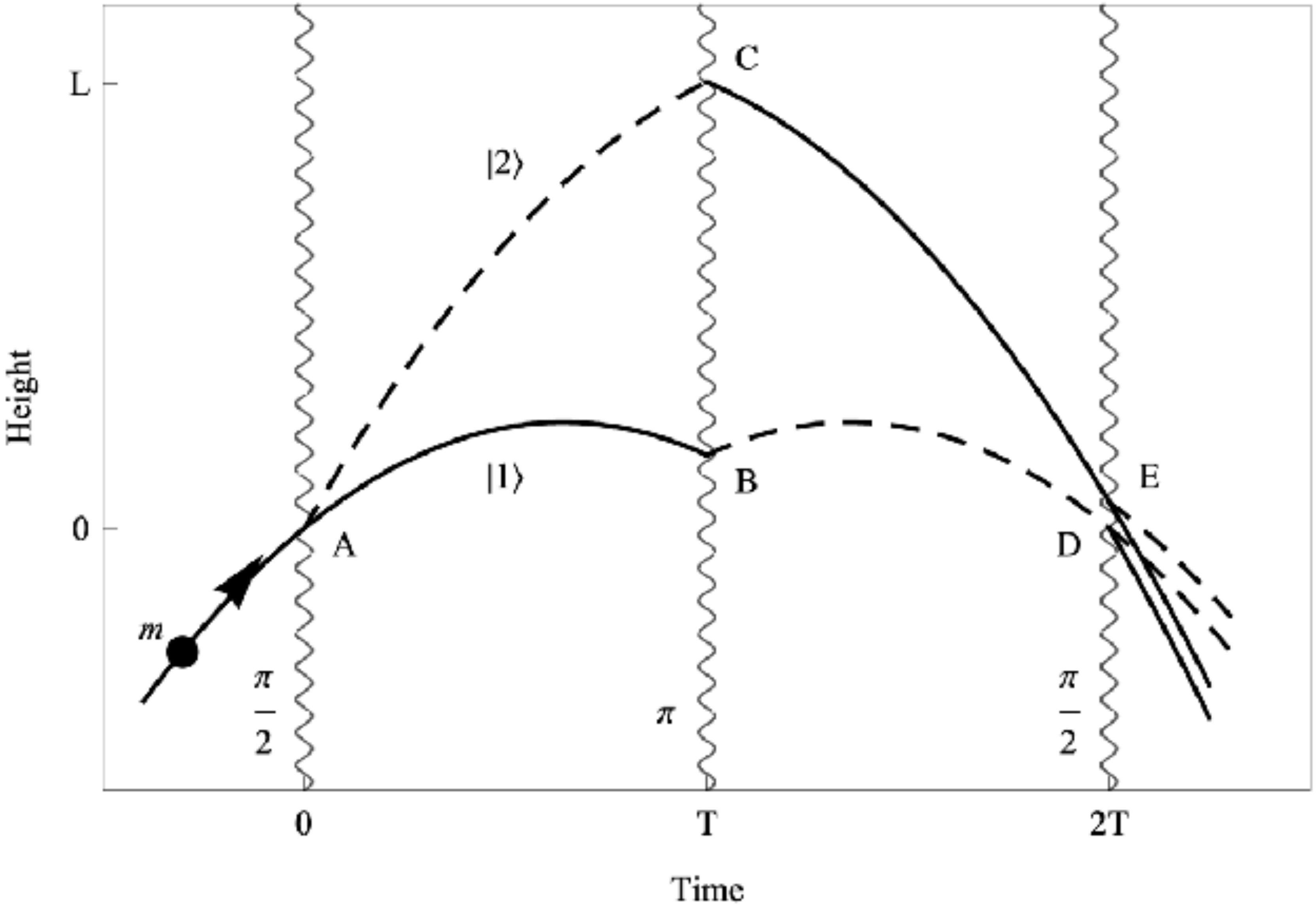
Michelson interferometer



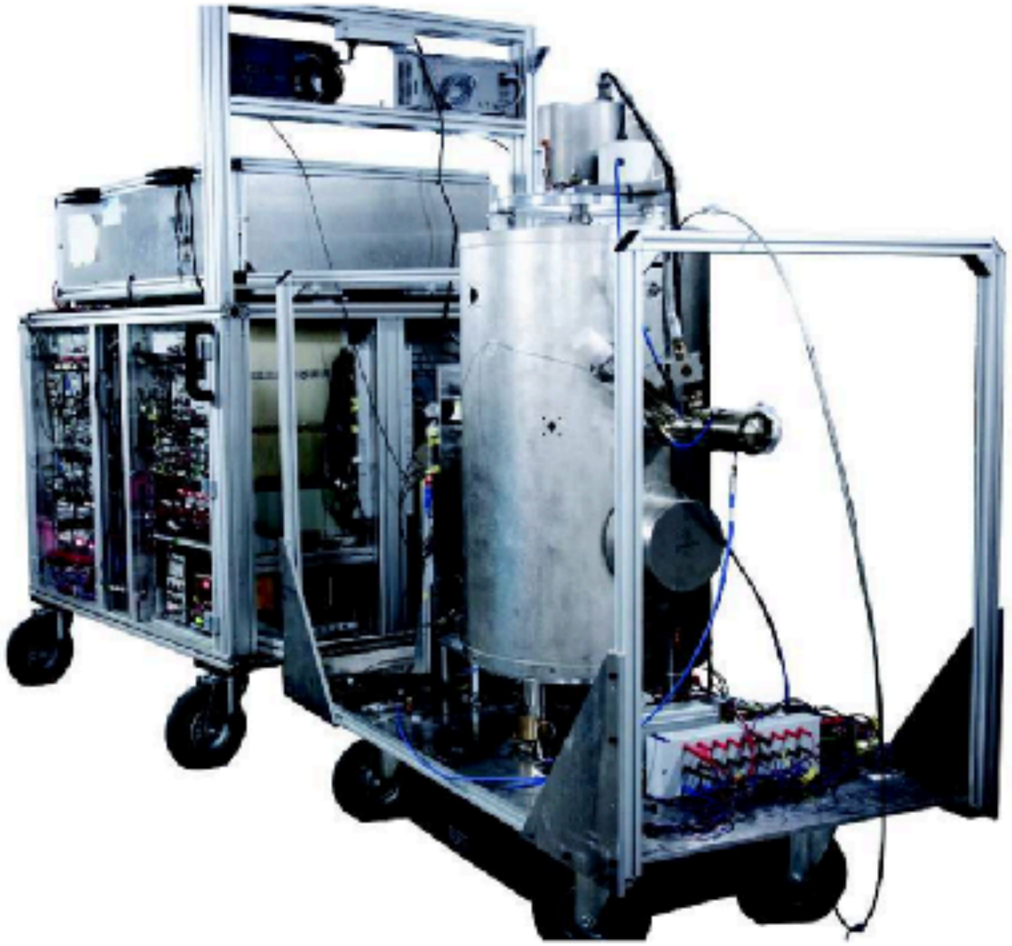
Mach–Zehnder interferometer

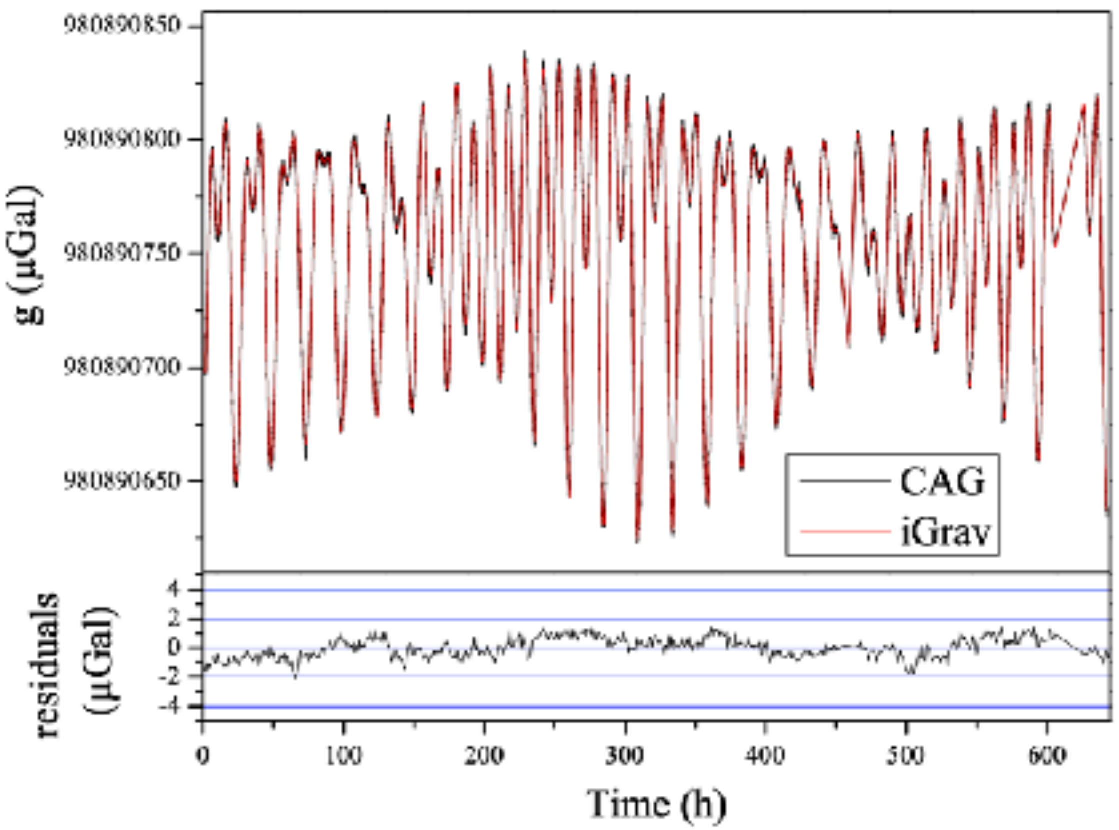
Examples of Quantum Sensing: Atom Interferometer





$$\phi = \mathbf{k}_{\rm eff} \cdot \mathbf{g} T^2$$





The LNE-SYRTE (France) cold atom gravimeter

$$1\ \text{Gal} = 10^{-2}\ \text{m/s}^2 \approx 10^{-3} g$$

Optical Magnetometer

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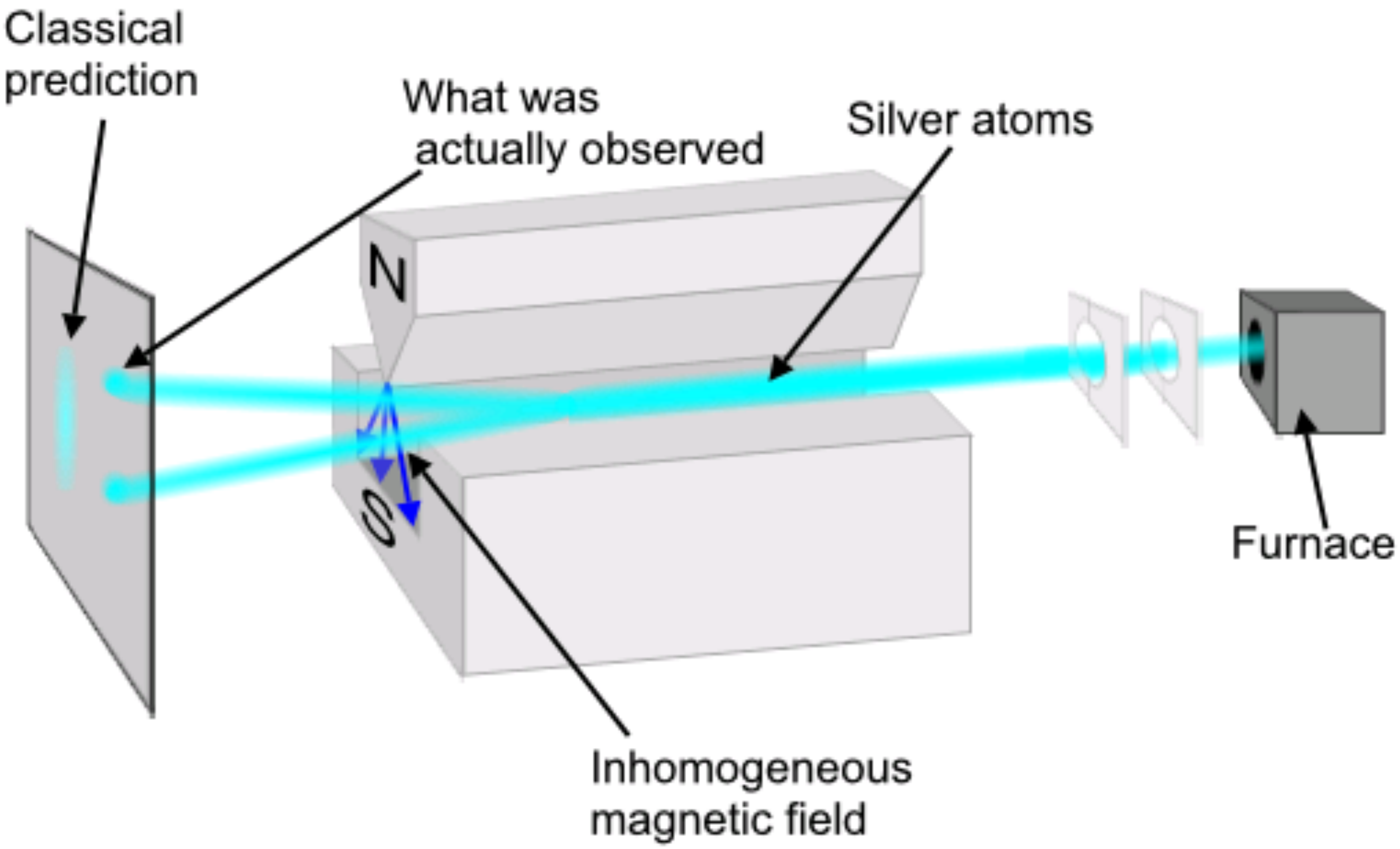
Stern-Gerlach Experiment



Otto Stern

Walter Gerlach

Spin is an intrinsic property of electron.

The Family of Magnetic Resonance


O. Stern
1943, phys.


I. Rabi
1944, phys.


F. Bloch
1952, phys.


R. Ernst
1991, chem.


K. Wüthrich
2002, chem.


P. Lauterbur
2003, med.

磁共振家族

自旋种类

◆核磁共振(NMR) ◆电子顺磁共振(ESR)

样品物态

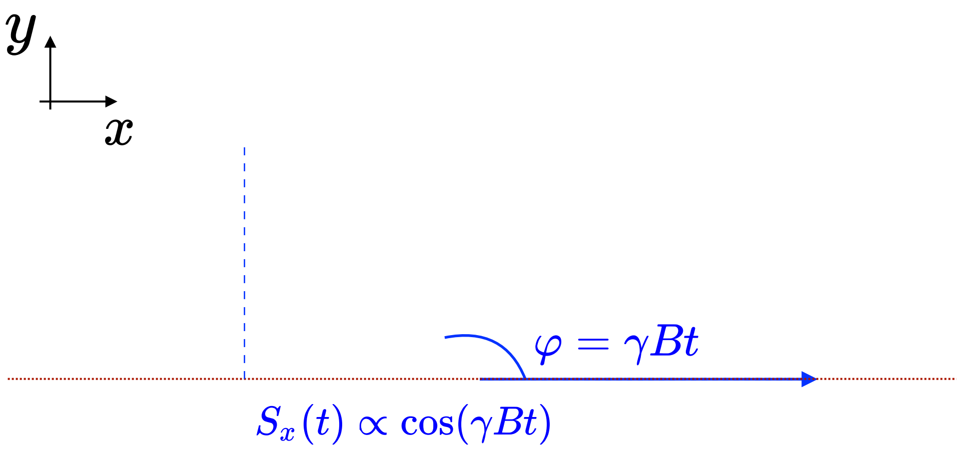
◆气态、液态磁共振 ◆固态磁共振

应用

◆分子结构 ◆组织成像
◆量子信息 ◆磁测量 ◆量子陀螺

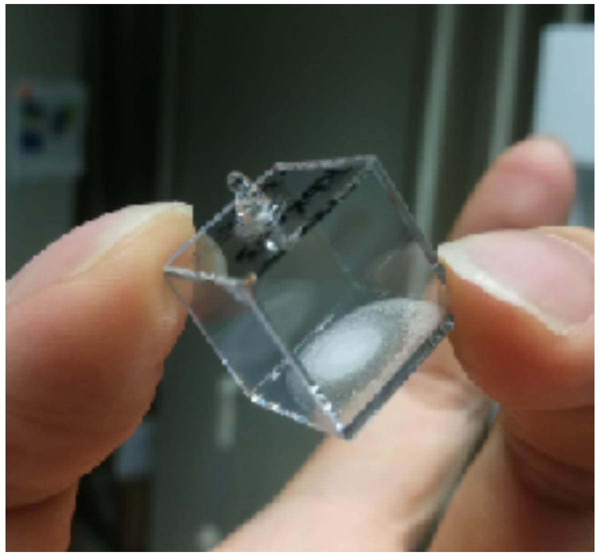
Examples of Quantum Sensing: Optical Magnetometer

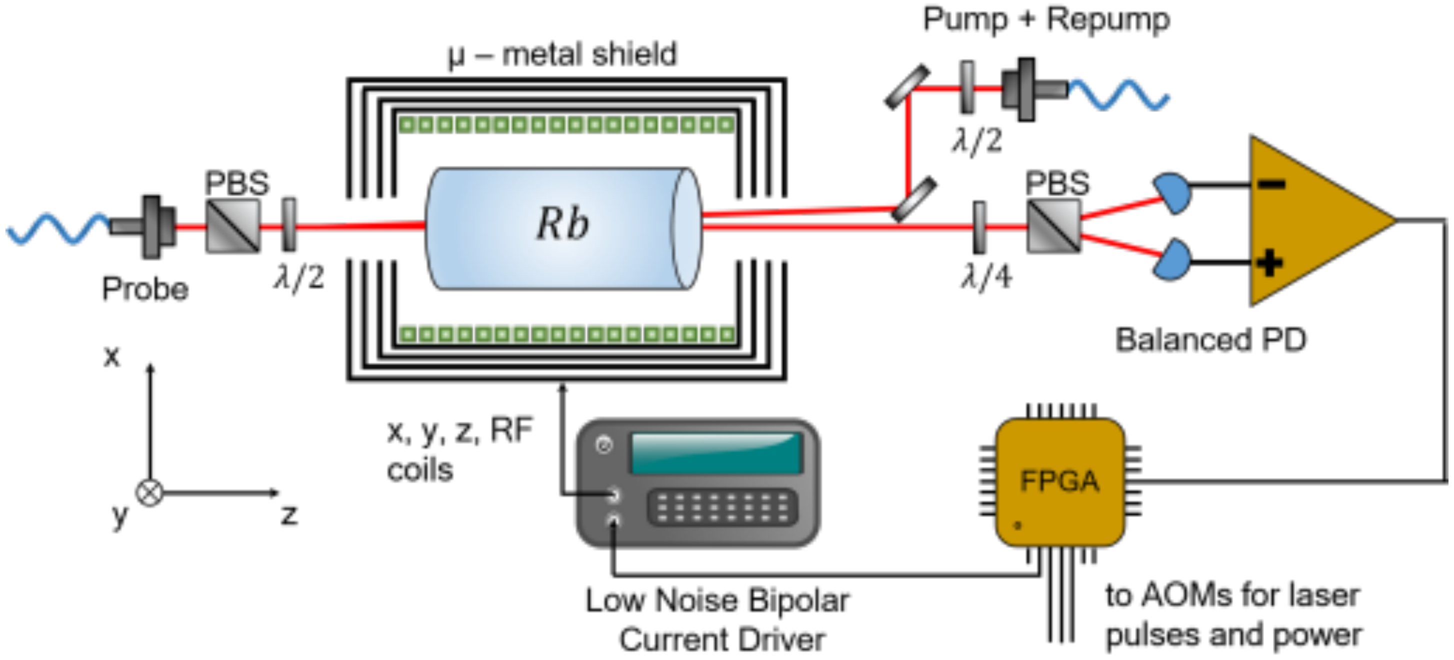
Spin precession in static field



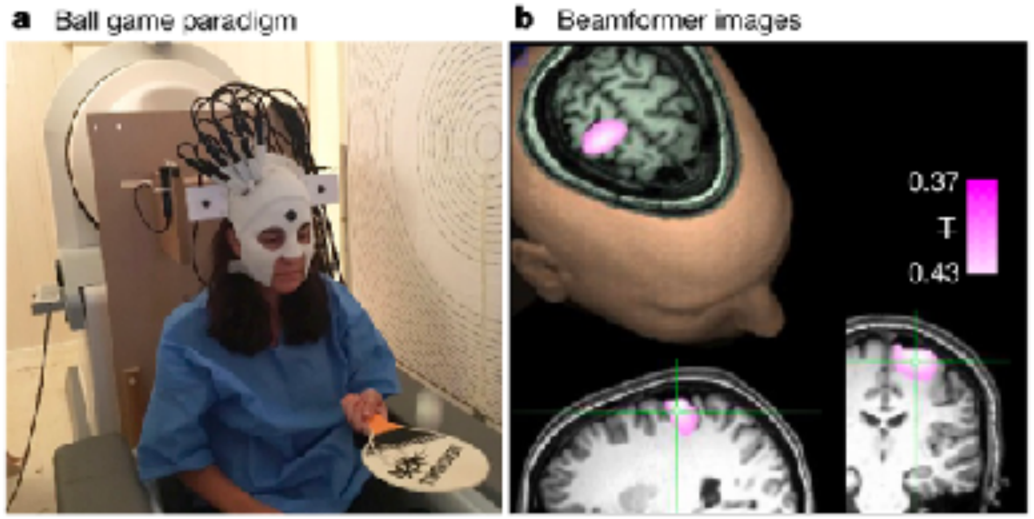
$$B = \frac{\varphi}{\gamma t} = \frac{\omega}{\gamma}$$

magnetic field is inferred from precession frequency





Examples of Quantum Sensing: Optical Magnetometer



Vapor Cell Magnetoencephalography (MEG)

脑磁图

Boto, E. et al. Nature **555**, 657 (2018).

Optical Magnetometer: Artificial Atoms in Solids

Diamond 4 C's

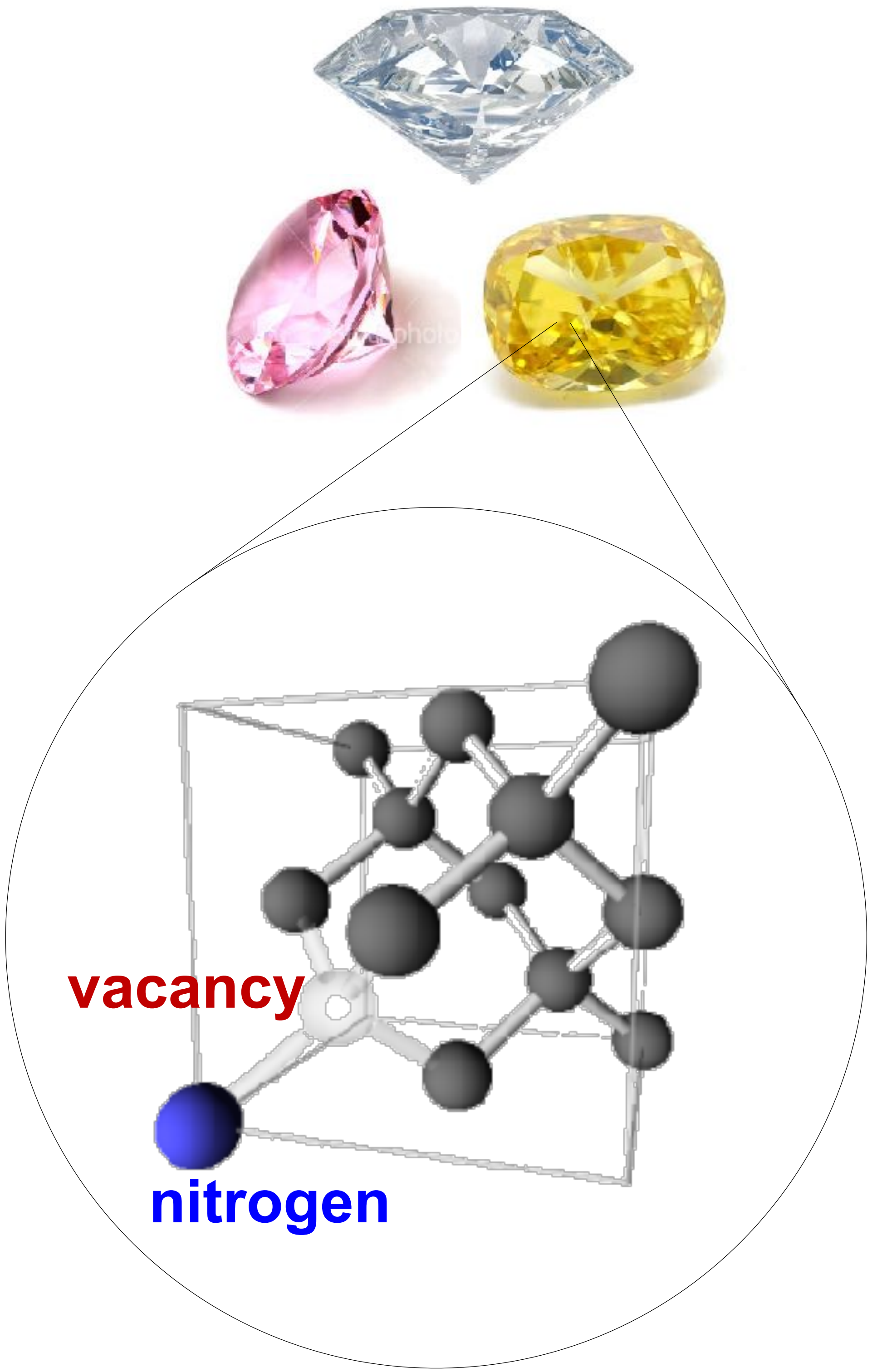
Carat

Clarity

Cut

Color

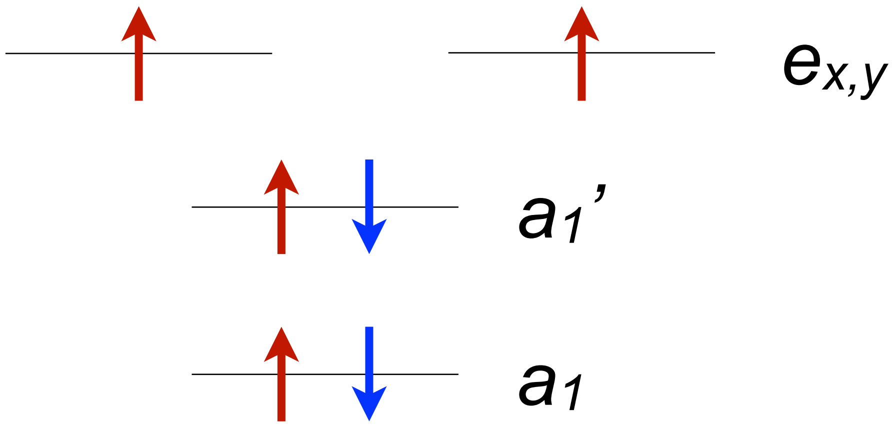
Optical Magnetometer: Artificial Atoms in Solids



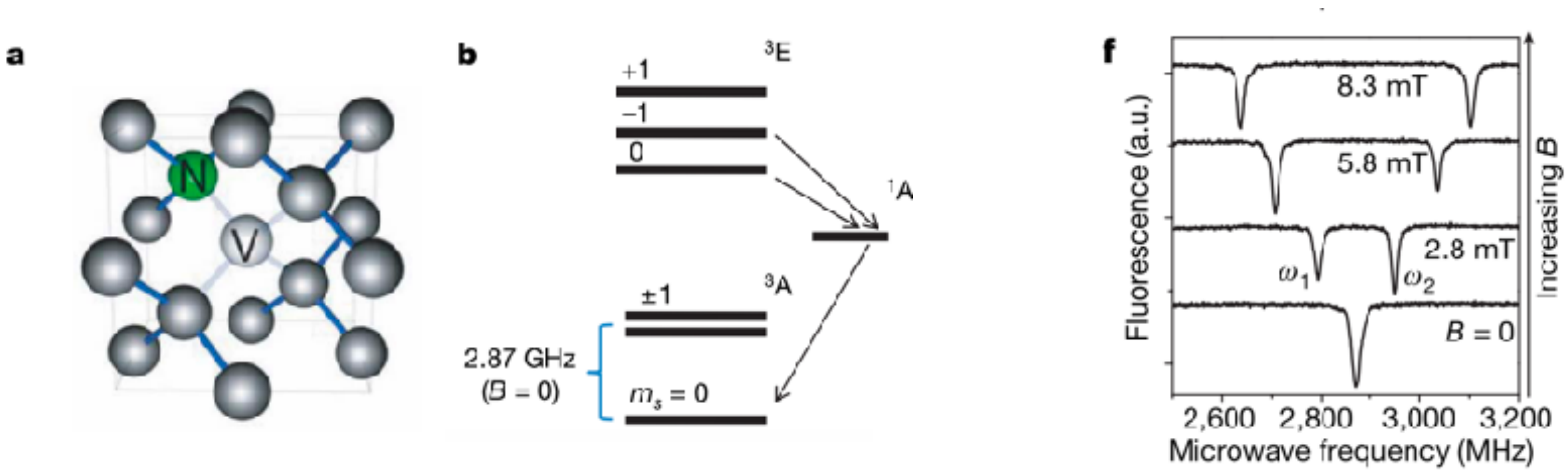
Negatively charged NV center

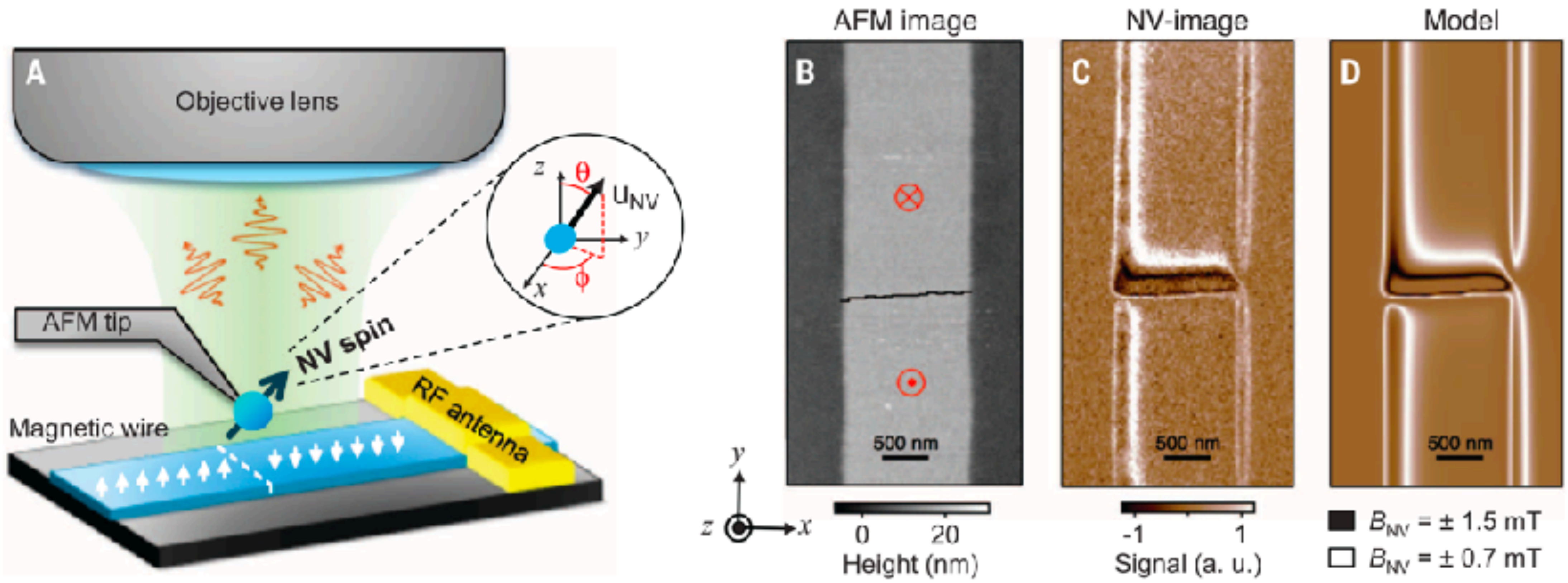
carbon sp^3 orbit	3 e
nitrogen orbit	2 e
negative charge	1 e
total	6 e

Ground state configuration



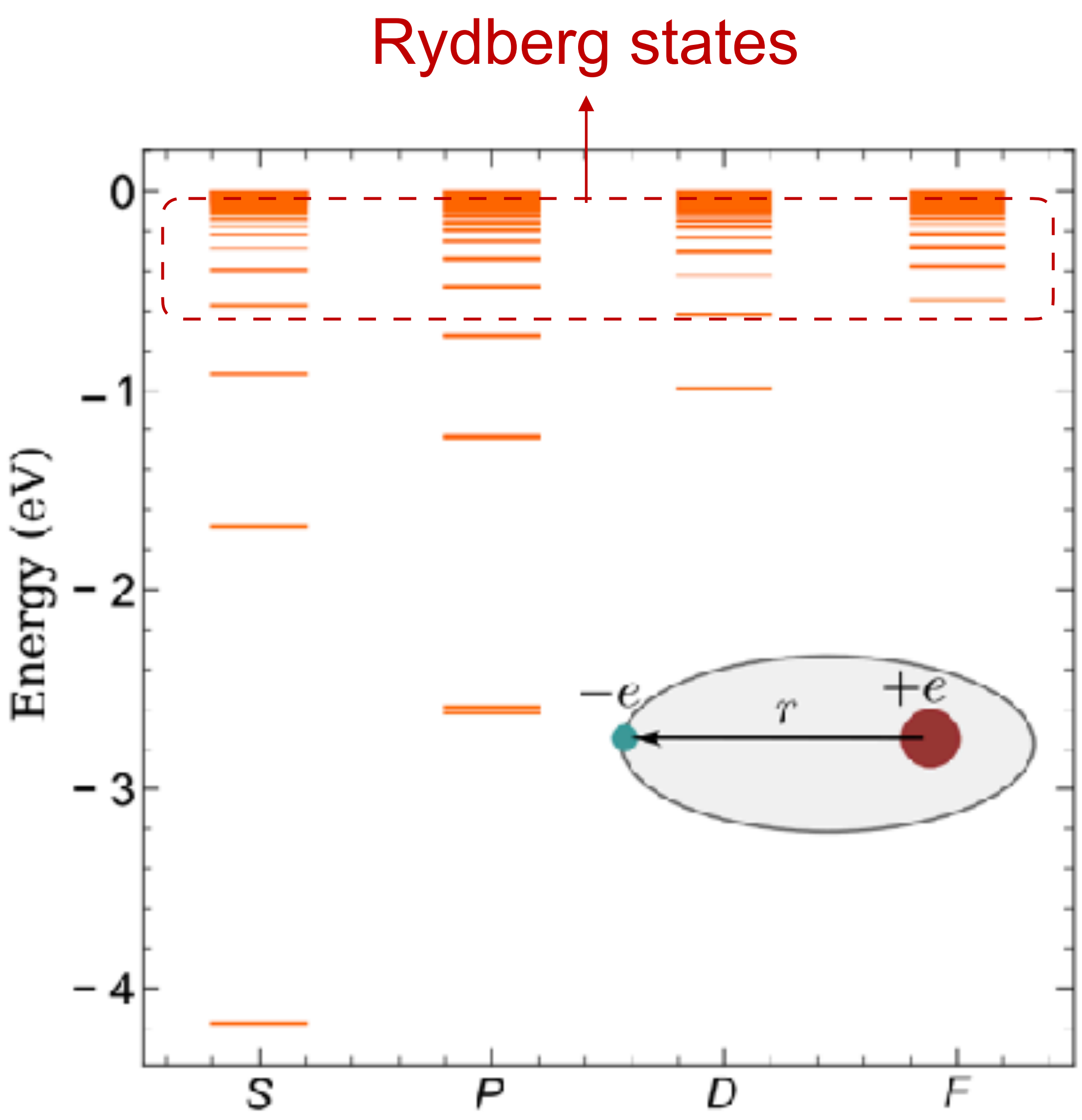
Optical Magnetometer: Artificial Atoms in Solids





Rydberg Atoms

Examples of Quantum Sensing: Rydberg Atoms



principle quantum number $n \gg 1$

$$E_n \propto -\frac{1}{n^2}$$

large size $\sim n^2$

large dipole $\sim n^2$

long lifetime $\sim n^5$

strong interaction $\sim n^4$

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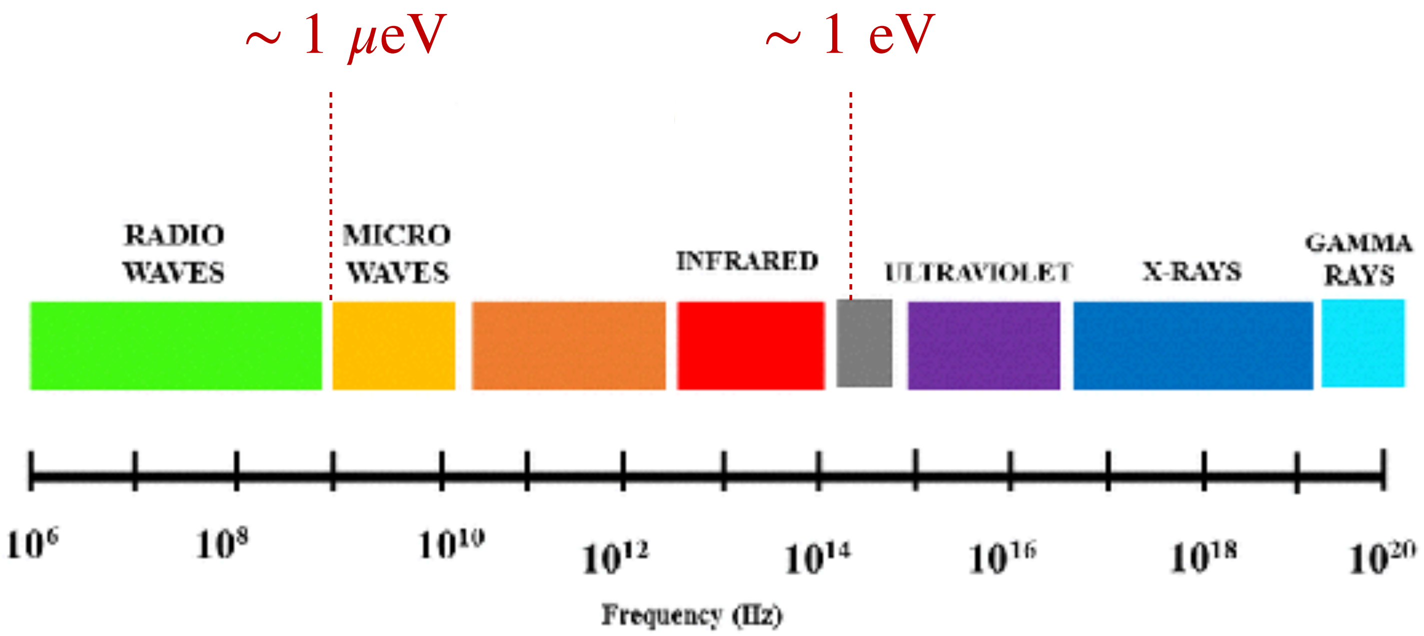
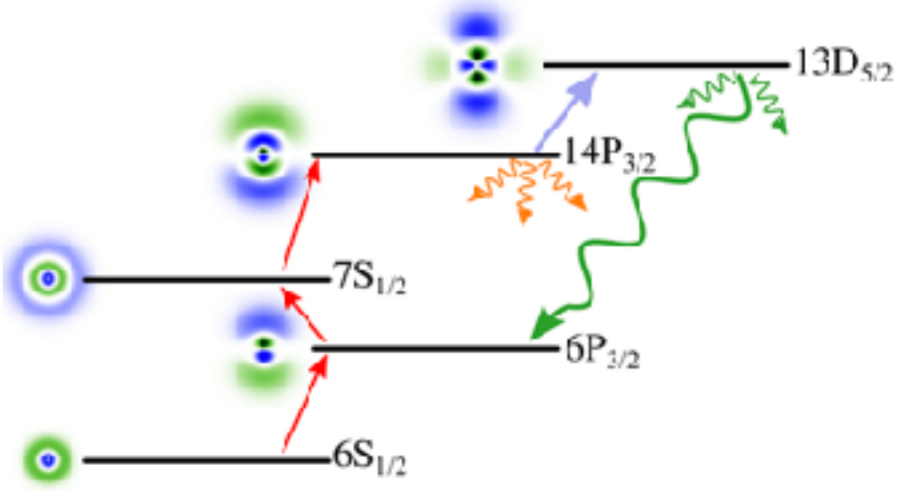
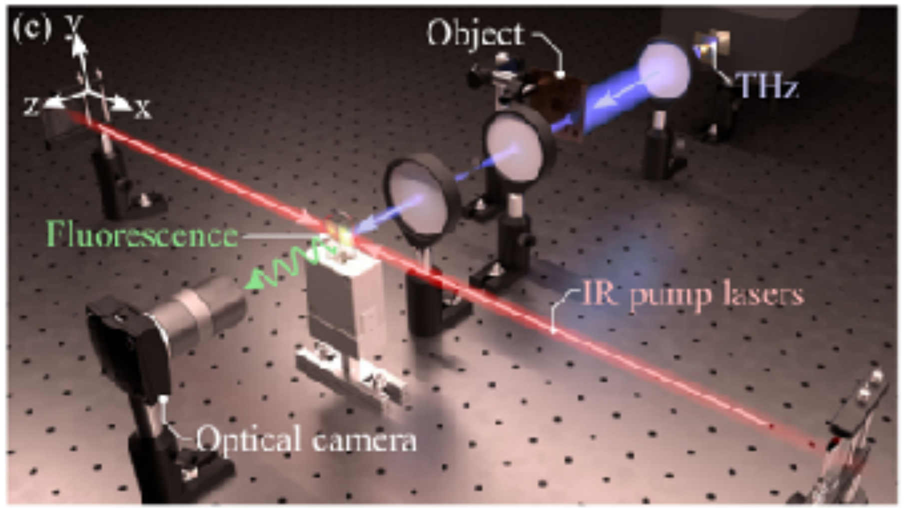
Examples of Quantum Sensing: Rydberg Atoms

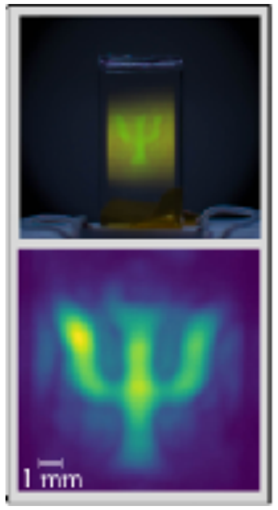


THz electric field sensing

by Rydberg atoms







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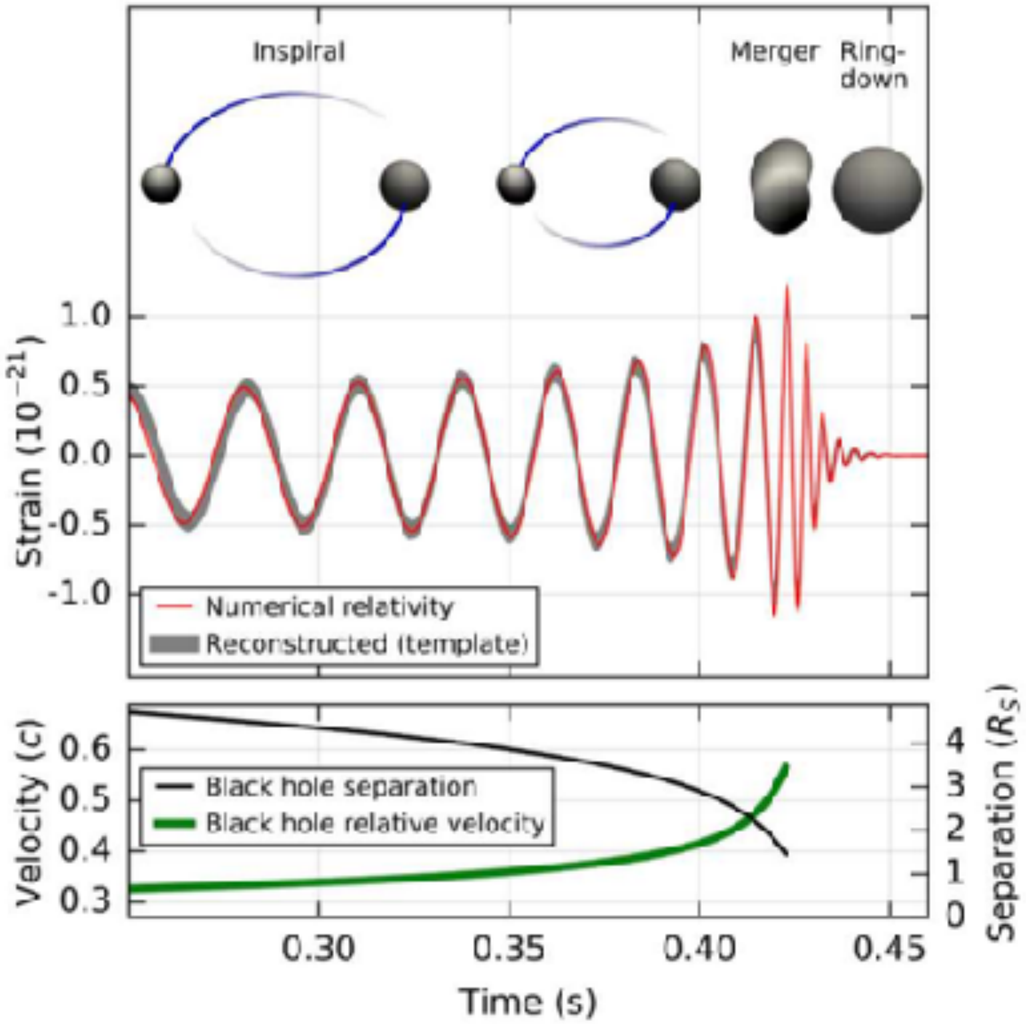
Quantum Resource

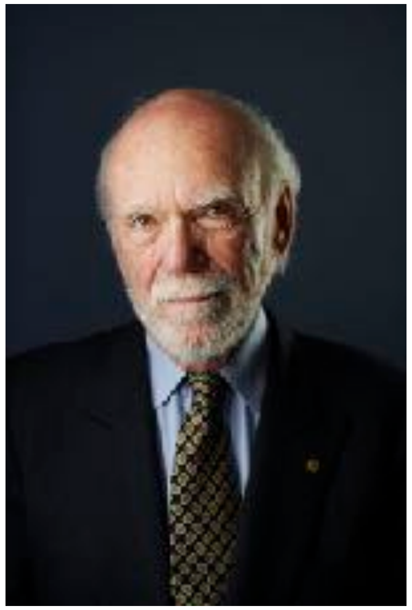
Examples of Quantum Sensing: Quantum Resource

First observation of gravitational waves
black hole merge event in 2016



Nobel Prize in Physics 2017

Rainer Weiss

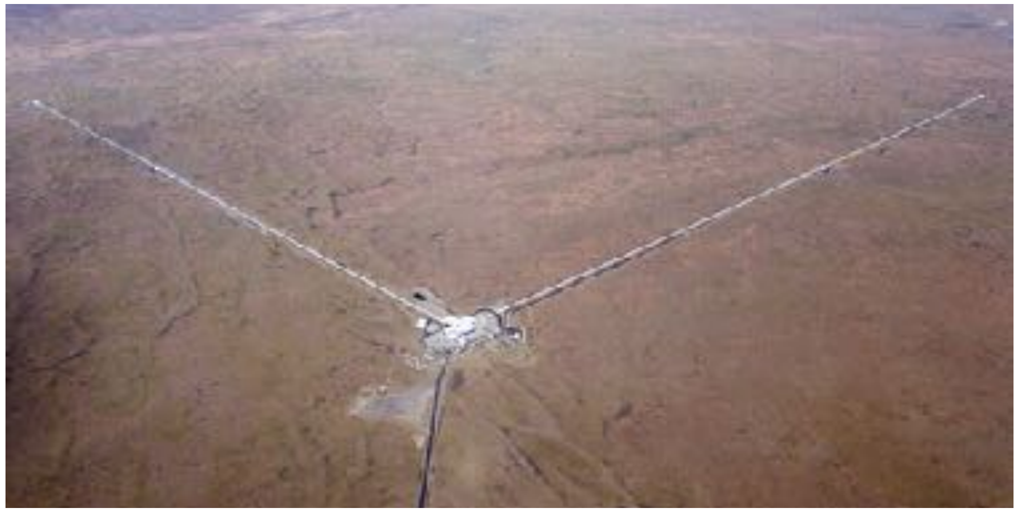


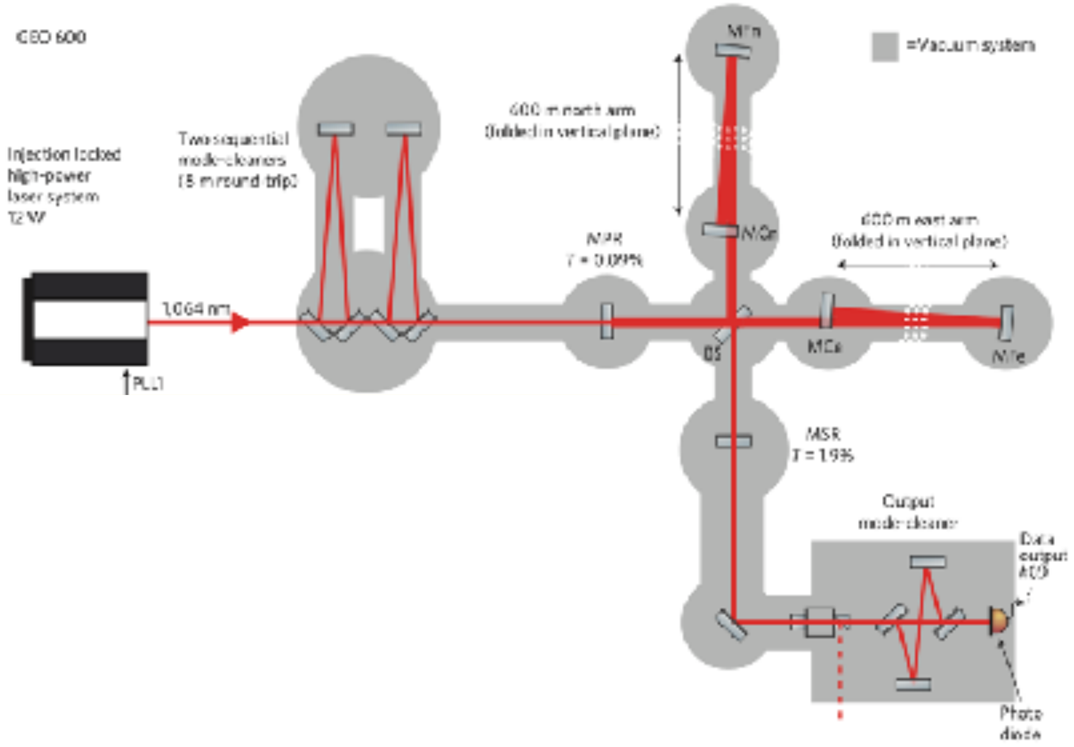
Barry C. Barish

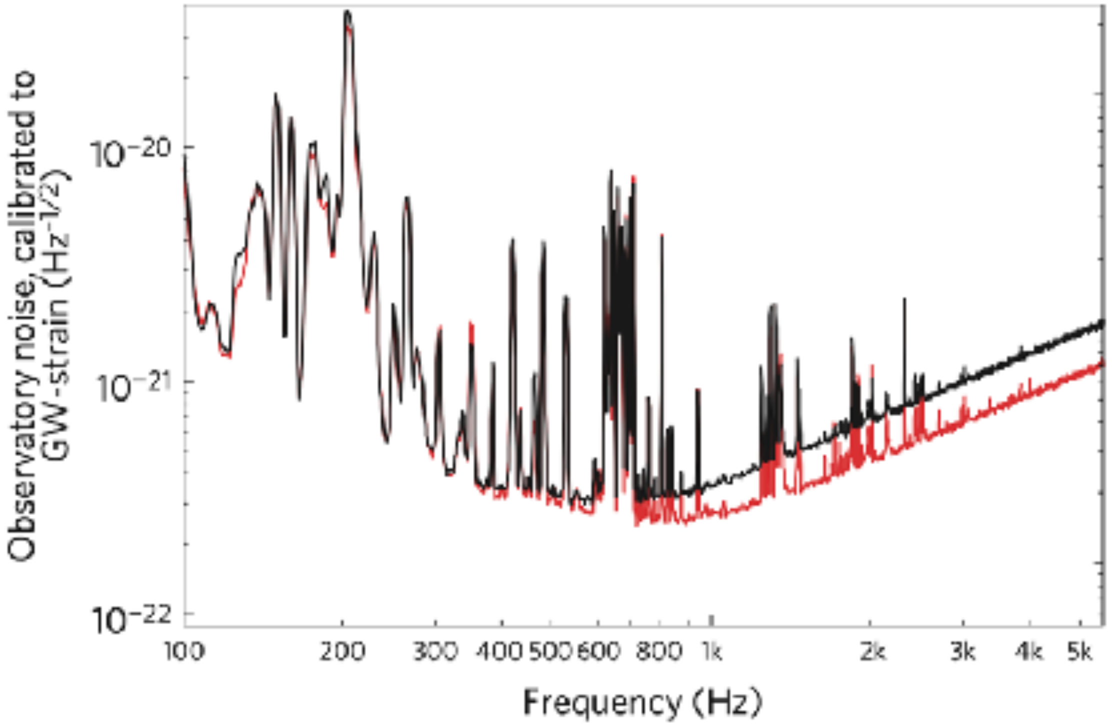
Kip S. Thorne

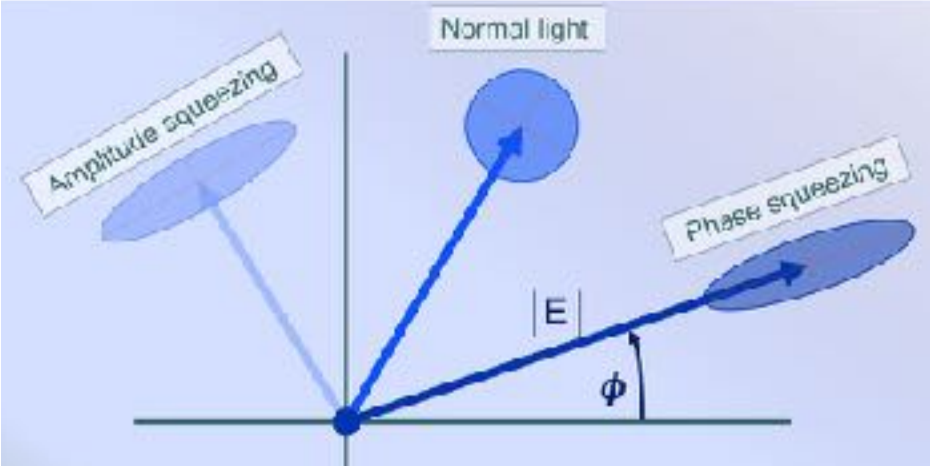
"for decisive contributions to the LIGO detector and the observation of gravitational waves."

Examples of Quantum Sensing: Quantum Resource



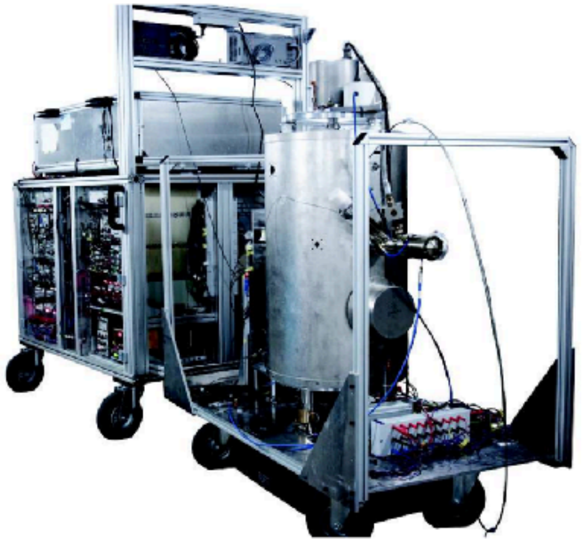


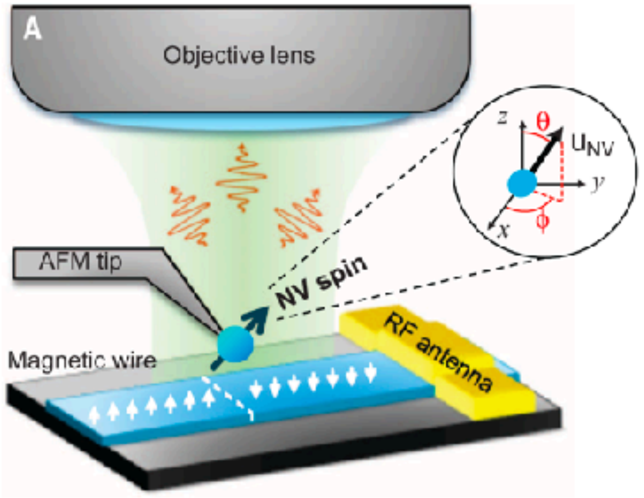


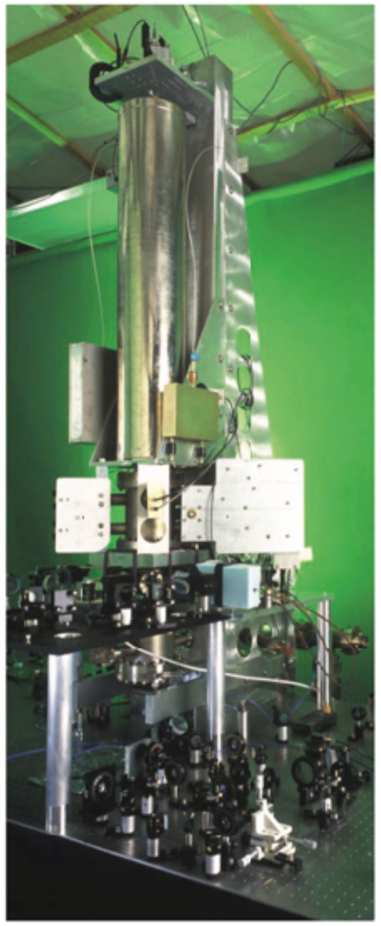


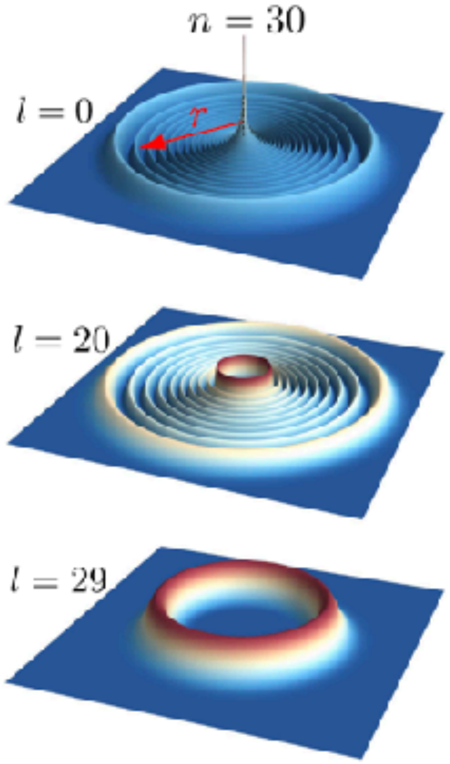
squeezed light: beyond the quantum shot-noise limit

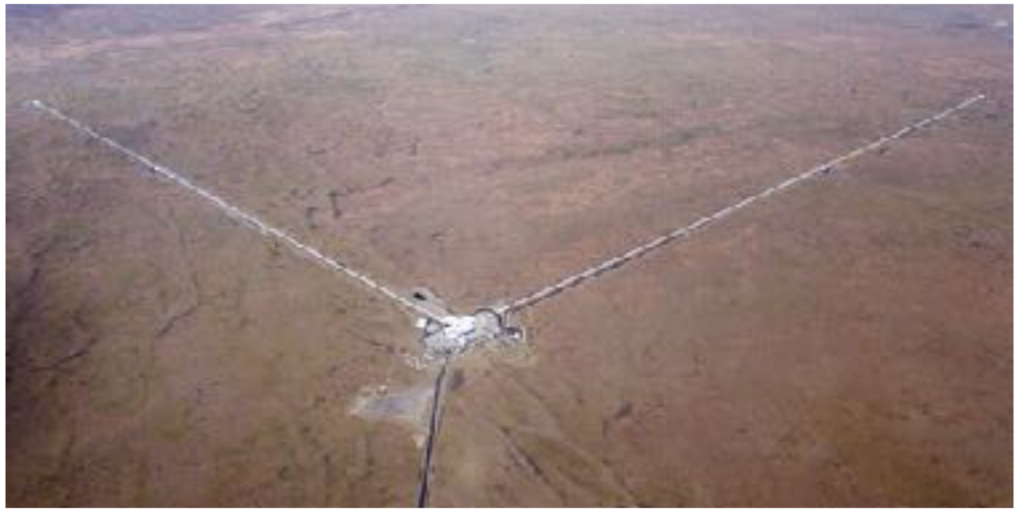
What is Quantum Sensing?











Quantum Sensing

1. What ?

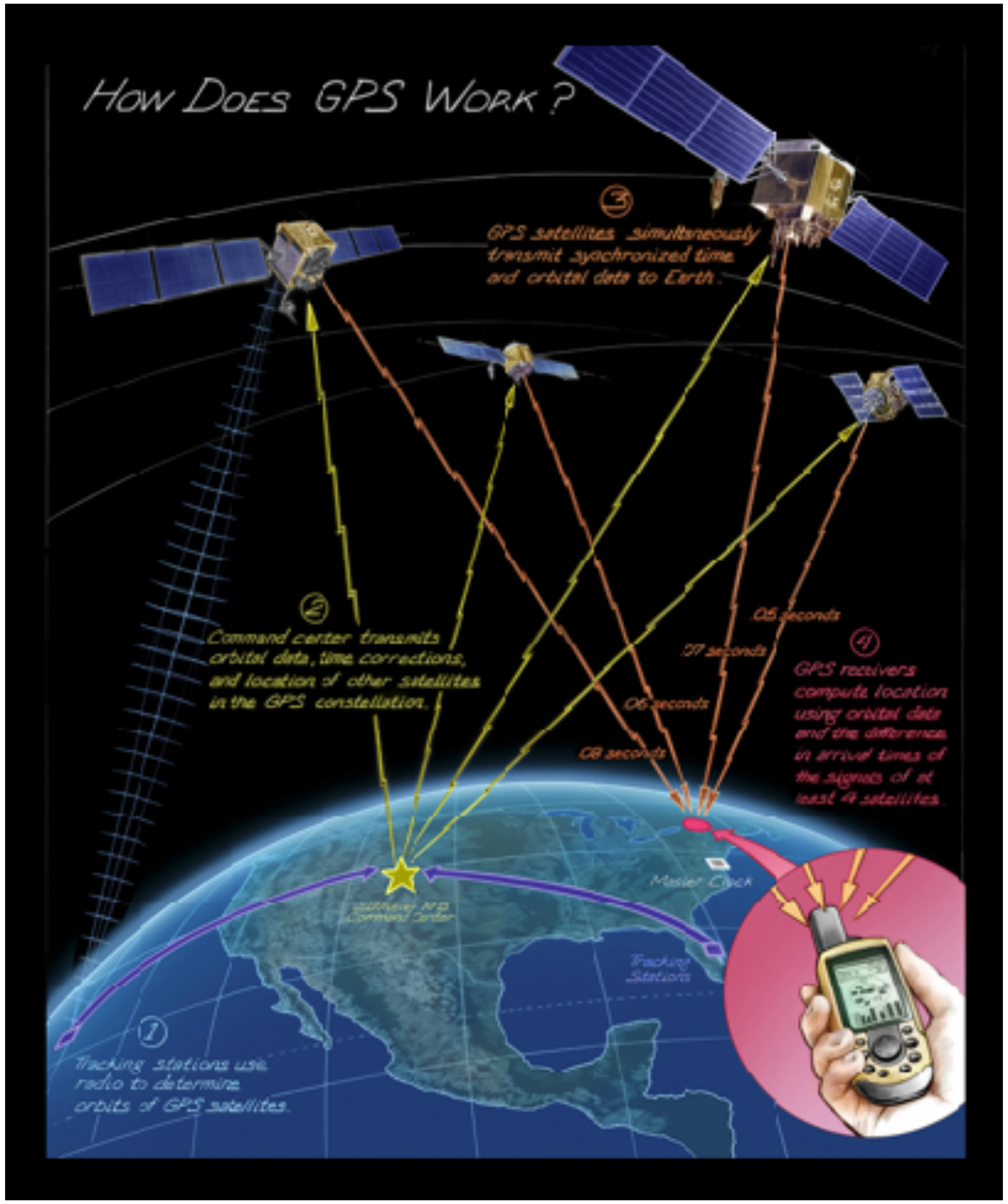
2. Why ?

3. How ?

CSRC

Improving Measurement Precision

CSRC

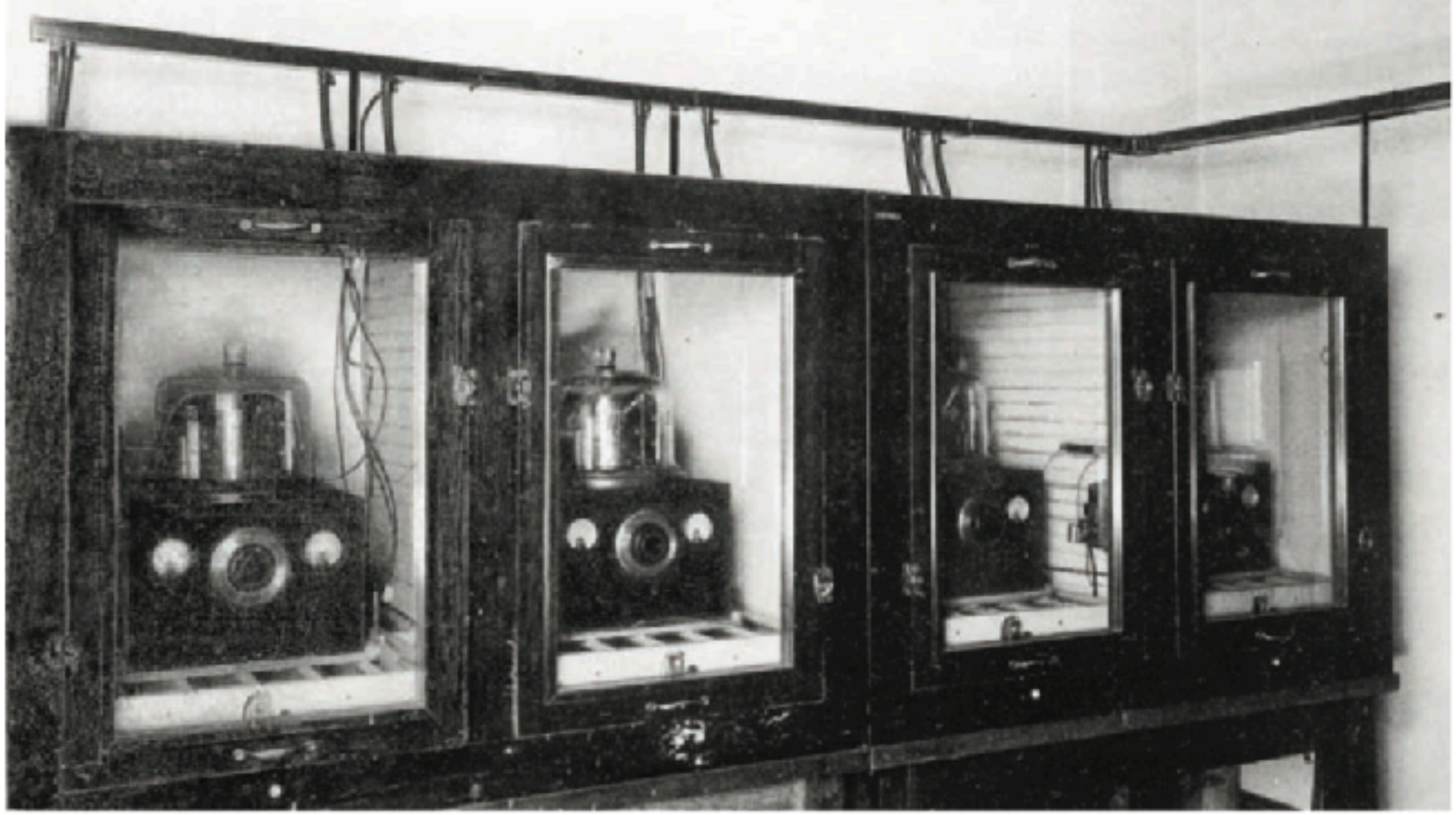


1. determine orbits of GPS satellites.
2. transmits orbital data, time corrections, and location of other satellites
3. transmit **synchronized time and orbital data** to Earth.
4. GPS receivers compute location using **orbital data and the difference in arrival times of the signals of at least 4 satellites**

Accurate time (frequency) measurement is crucial!

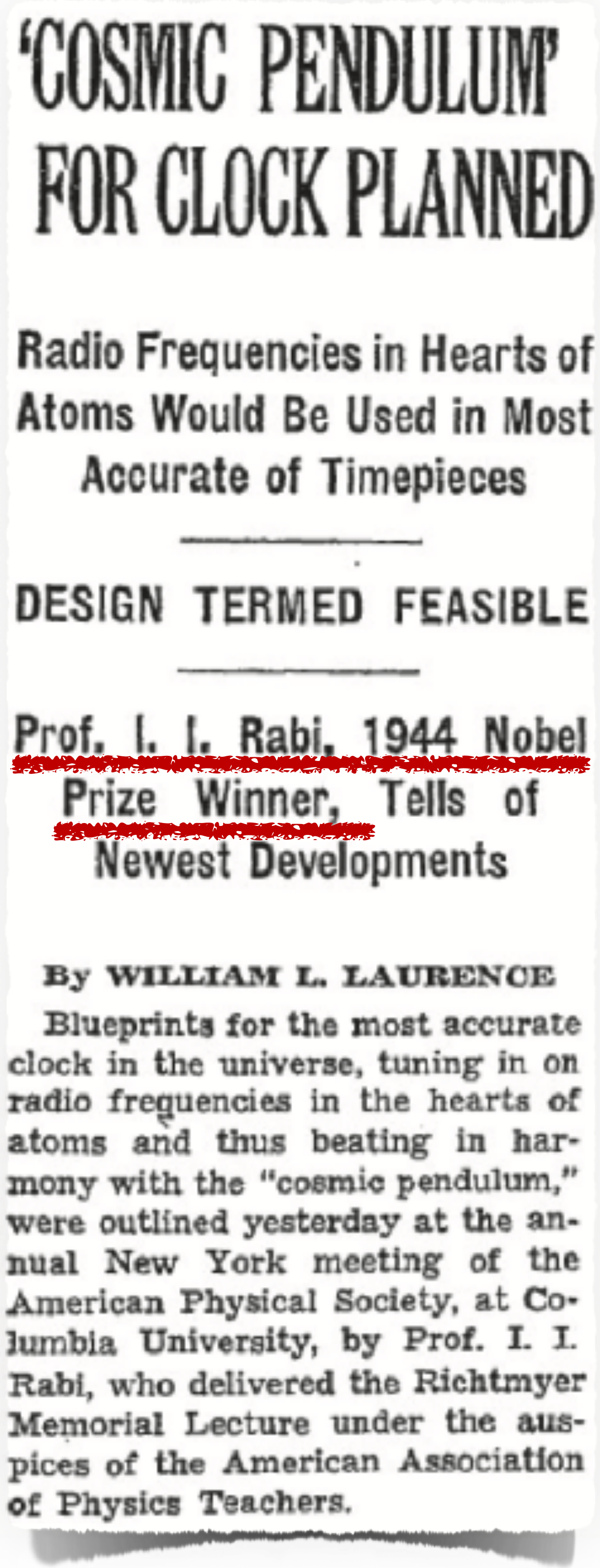
Early time standard before atomic clock

pendulum clock
1904-1929
$\sim 1 \times 10^{-7}$



quartz crystal oscillators, 1929

best accuracy 1×10^{-9} (1950)



'COSMIC PENDULUM' FOR CLOCK PLANNED

Radio Frequencies in Hearts of Atoms Would Be Used in Most Accurate of Timepieces

DESIGN TERMED FEASIBLE

Prof. I. I. Rabi, 1944 Nobel Prize Winner, Tells of Newest Developments

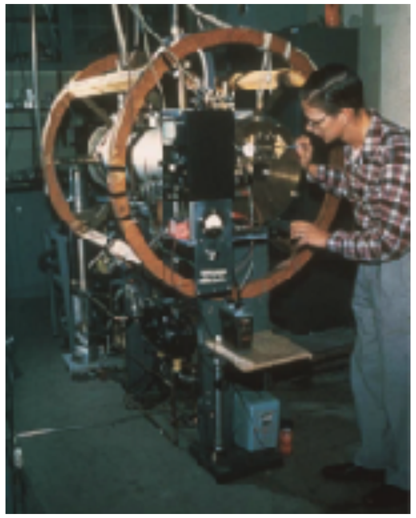
By WILLIAM L. LAURENCE

Blueprints for the most accurate clock in the universe, tuning in on radio frequencies in the hearts of atoms and thus beating in harmony with the "cosmic pendulum," were outlined yesterday at the annual New York meeting of the American Physical Society, at Columbia University, by Prof. I. I. Rabi, who delivered the Richtmyer Memorial Lecture under the auspices of the American Association of Physics Teachers.

The first atomic frequency standard, based on the ammonia molecule, with accuracy $\sim 2 \times 10^{-8}$ in 1949

Harold Lyons (right) & Edward Condon (left)

National Primary Frequency Standards



NBS-1, 1959

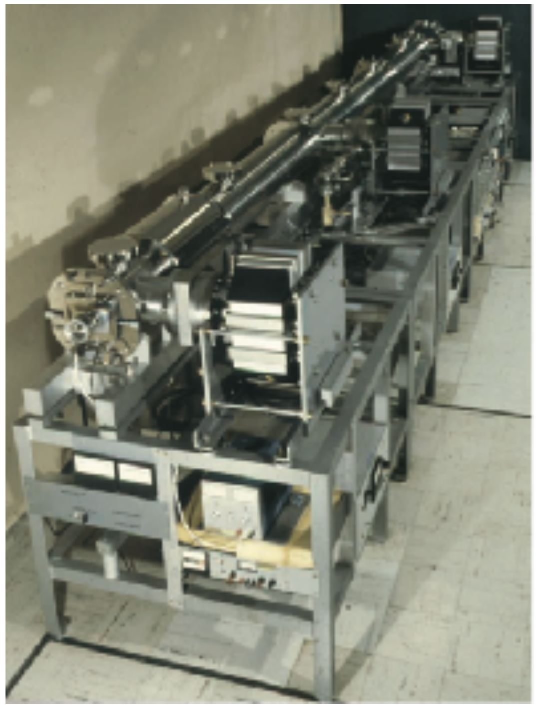
1×10^{-11}

NBS-2, 1960

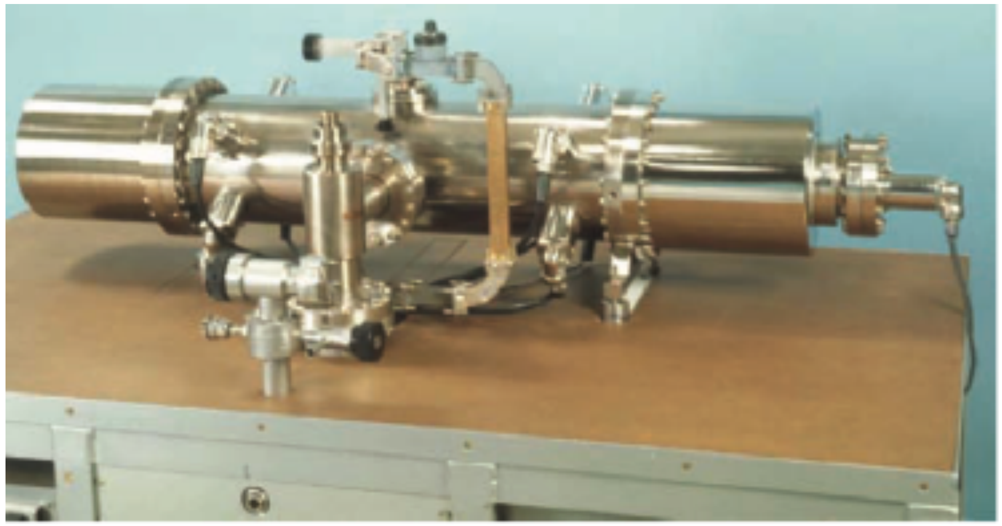
8×10^{-12}

NBS-3, 1963

5×10^{-13}

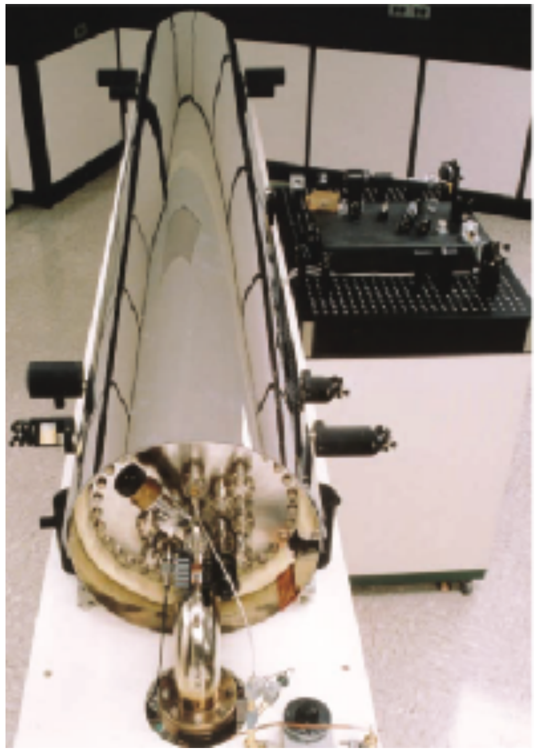


NBS-5, 1972 2×10^{-13}



NBS-4 3×10^{-13}

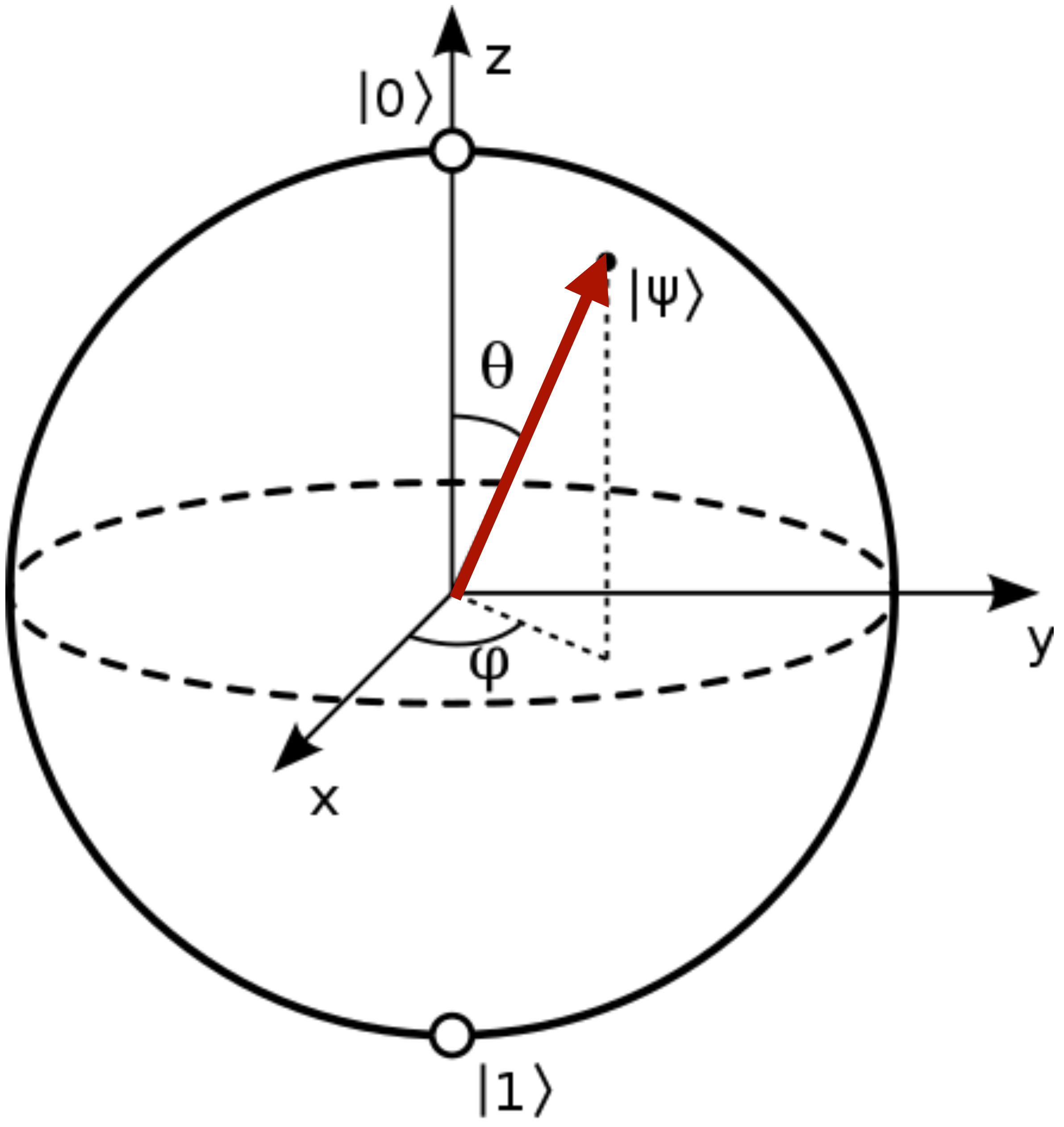
NBS-6, 1975 8×10^{-14}



NIST-7, 1993 5×10^{-15}

NBS = National Bureau of Standards
NIST = National Institute of Standards and Technology

CSRC



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$= \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\varphi}|1\rangle$$

- normalization condition

$$|\alpha|^2 + |\beta|^2 = 1$$

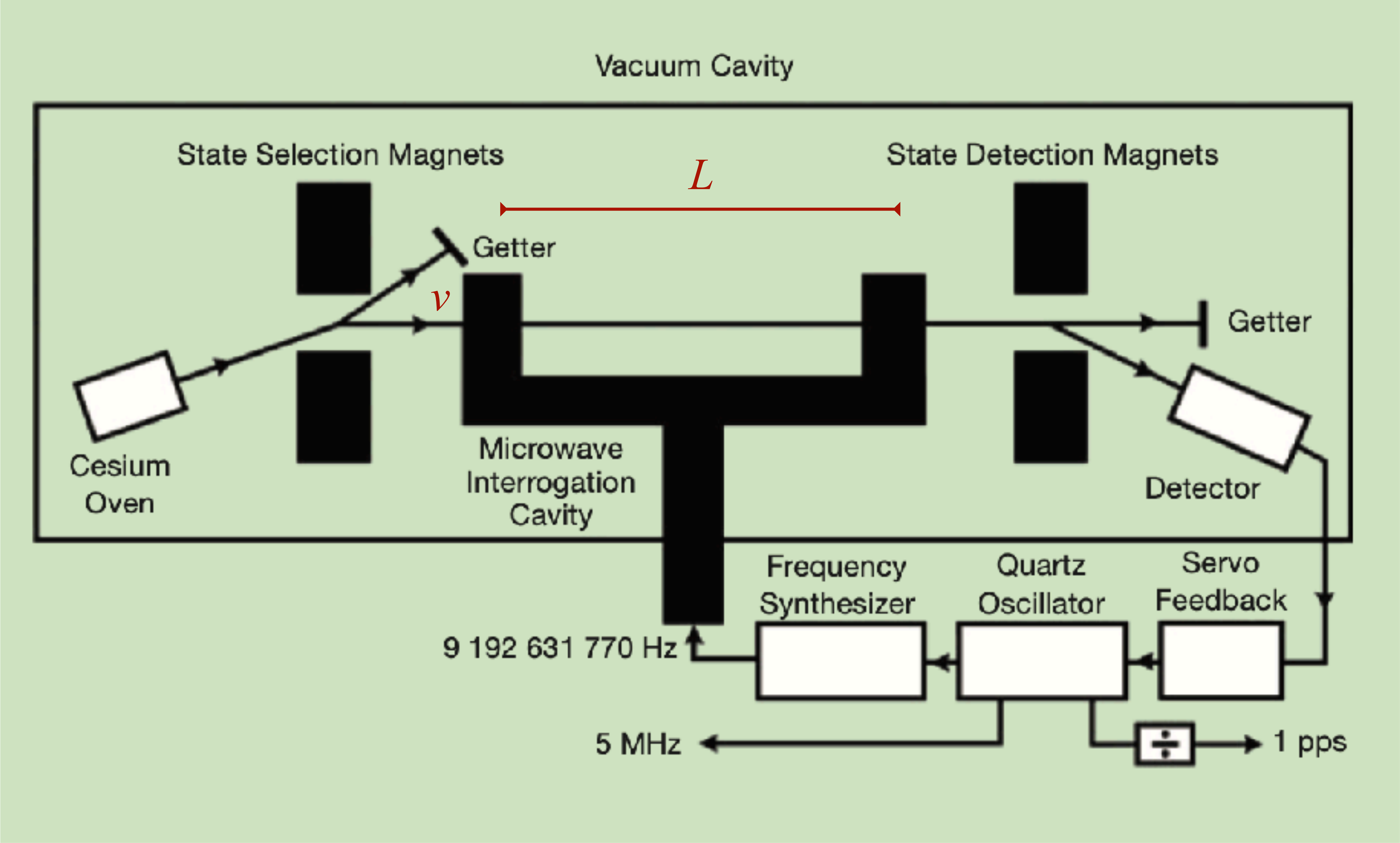
- global phase does not change $|\psi\rangle$

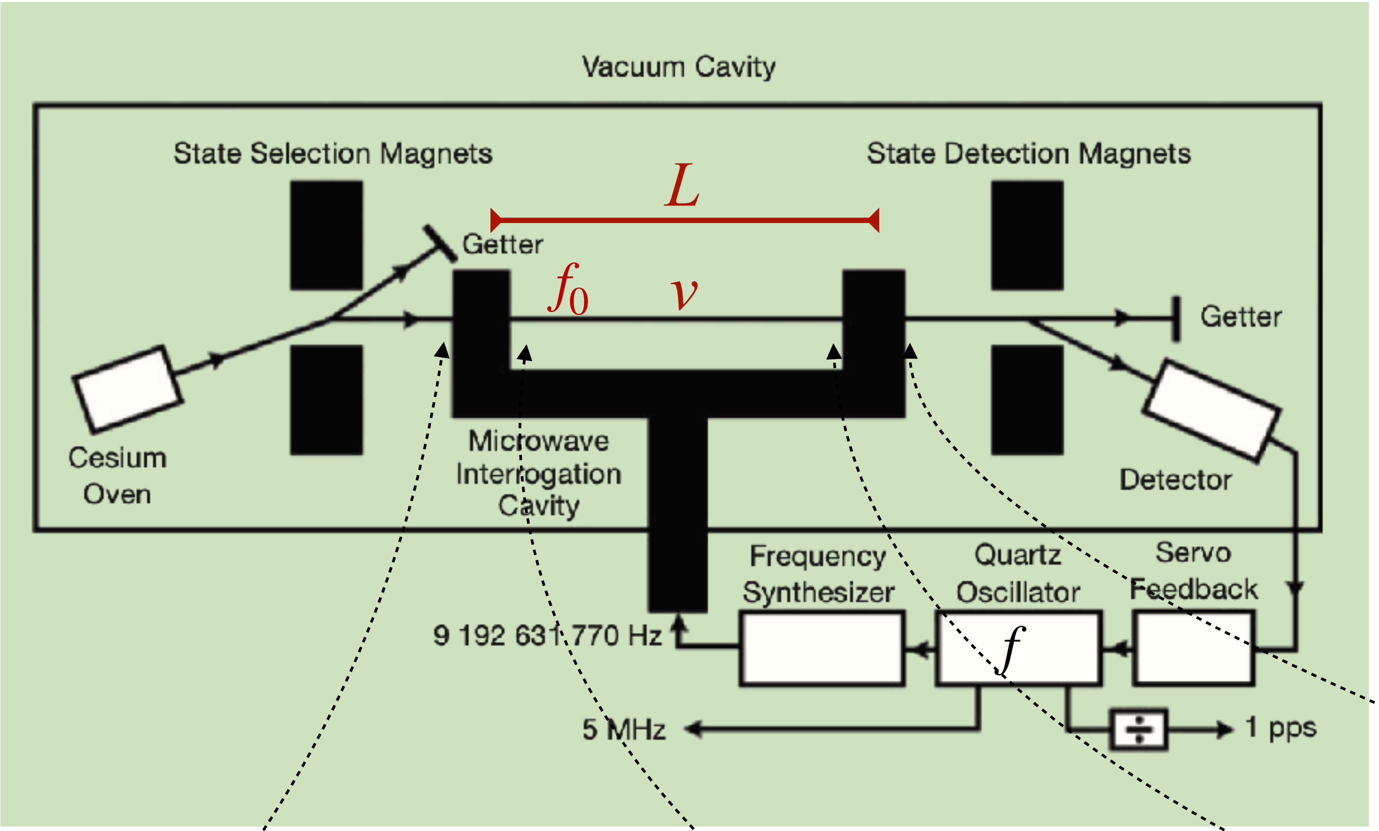
$$\langle\sigma_x\rangle = \sin\theta\cos\varphi$$

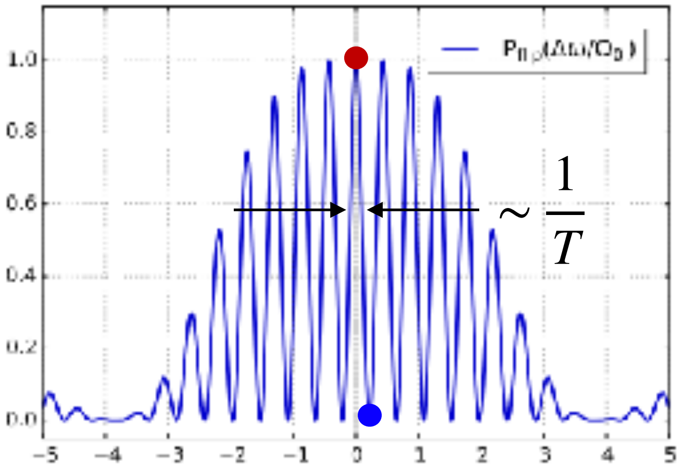
$$\langle\sigma_y\rangle = \sin\theta\sin\varphi$$

$$\langle\sigma_z\rangle = \cos\theta$$

CSRC

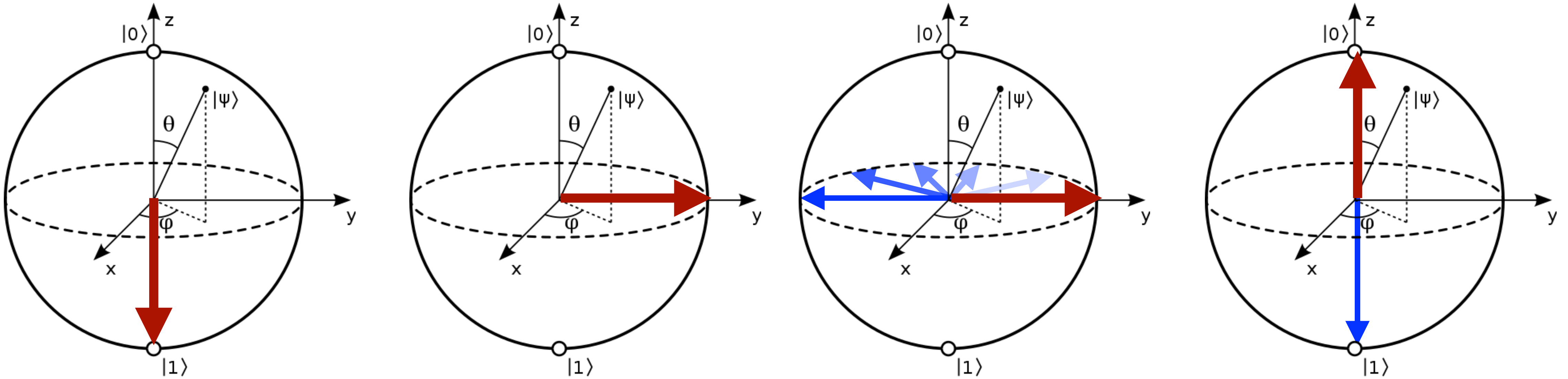

Vacuum Cavity
State Selection Magnets
L
State Detection Magnets
Getter
v
Getter
Cesium Oven
Microwave Interrogation Cavity
Detector
Frequency Synthesizer
Quartz Oscillator
Servo Feedback
9 192 631 770 Hz
5 MHz
÷
1 pps



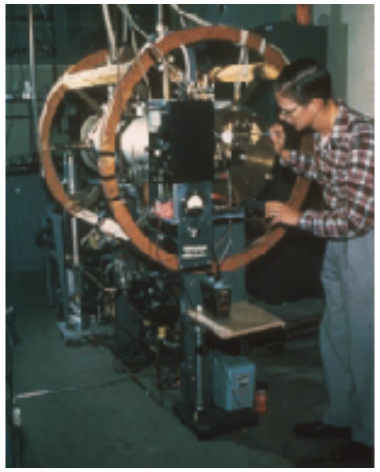


$$P_0(T,\Delta) = \cos^2\left(\frac{\Delta T}{2}\right) = \cos^2\left(\frac{\Delta L}{2v}\right)$$

$$\Delta = 2\pi(f - f_0)$$

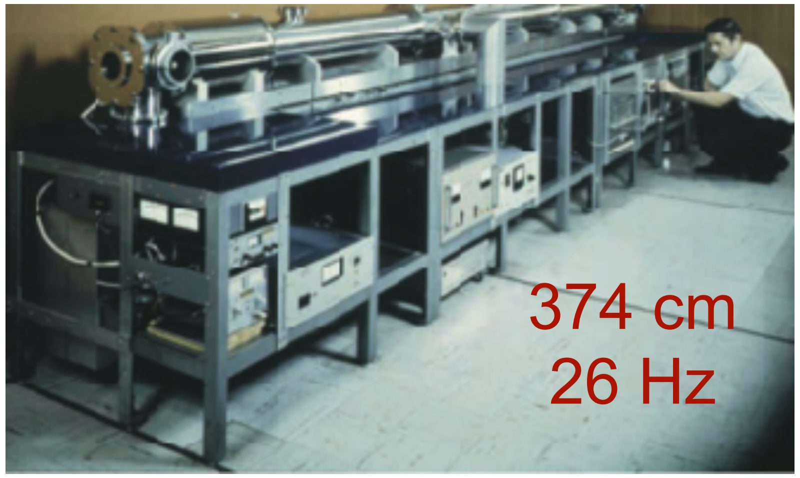


$t < 0$ $t = 0$ $t = T$ $t > T$



55 cm
300 Hz

NBS-1, 1959 1×10^{-11}

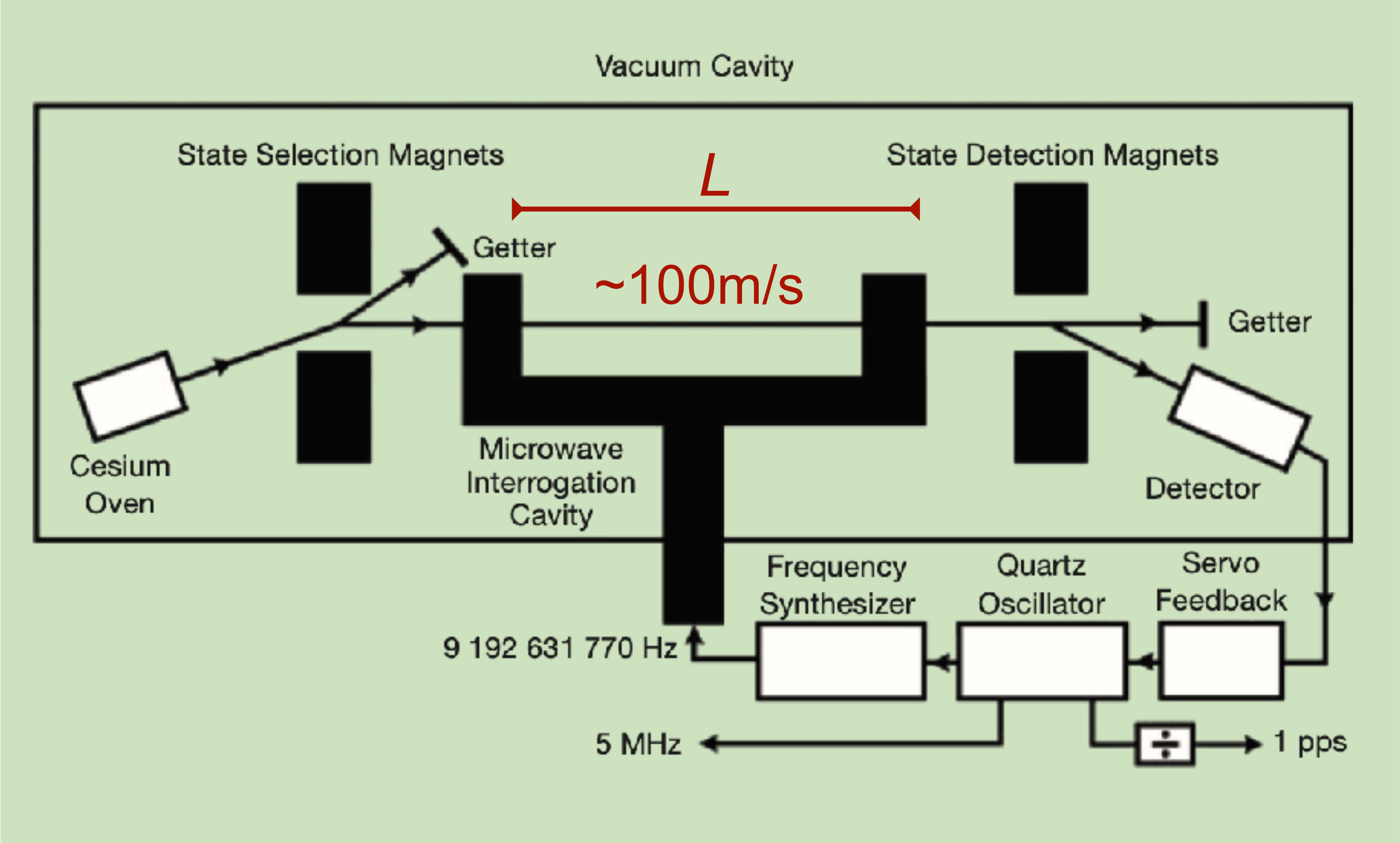


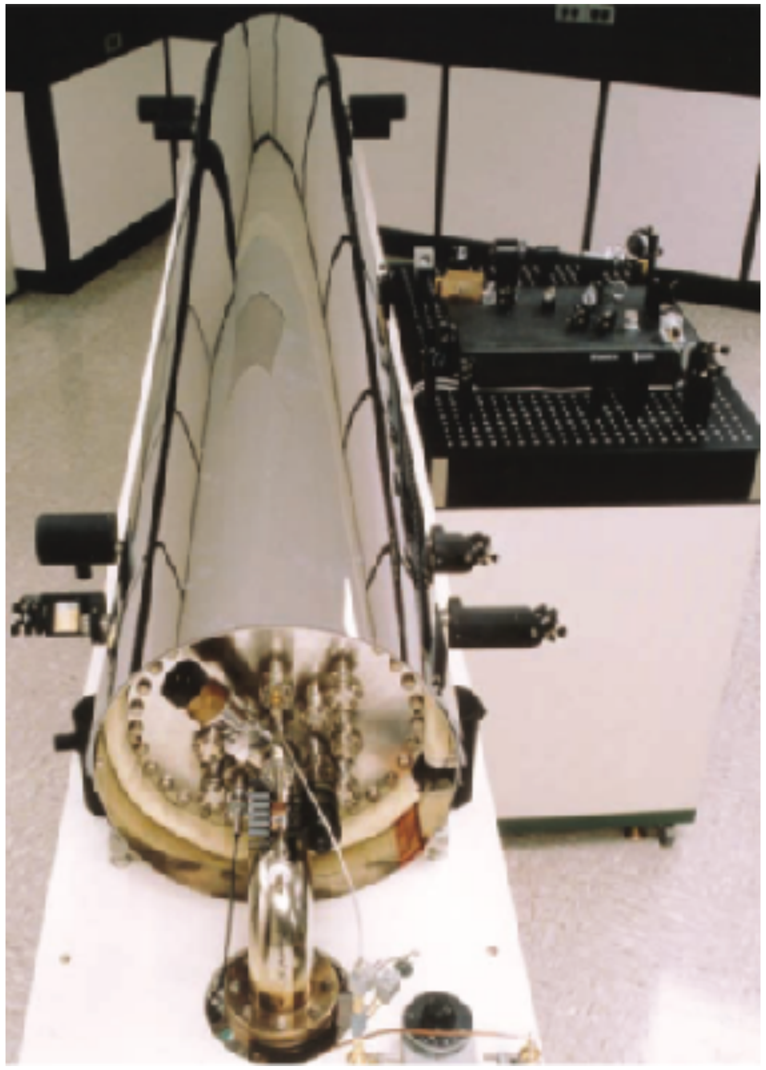
374 cm
26 Hz

NBS-6, 1975 8×10^{-14}

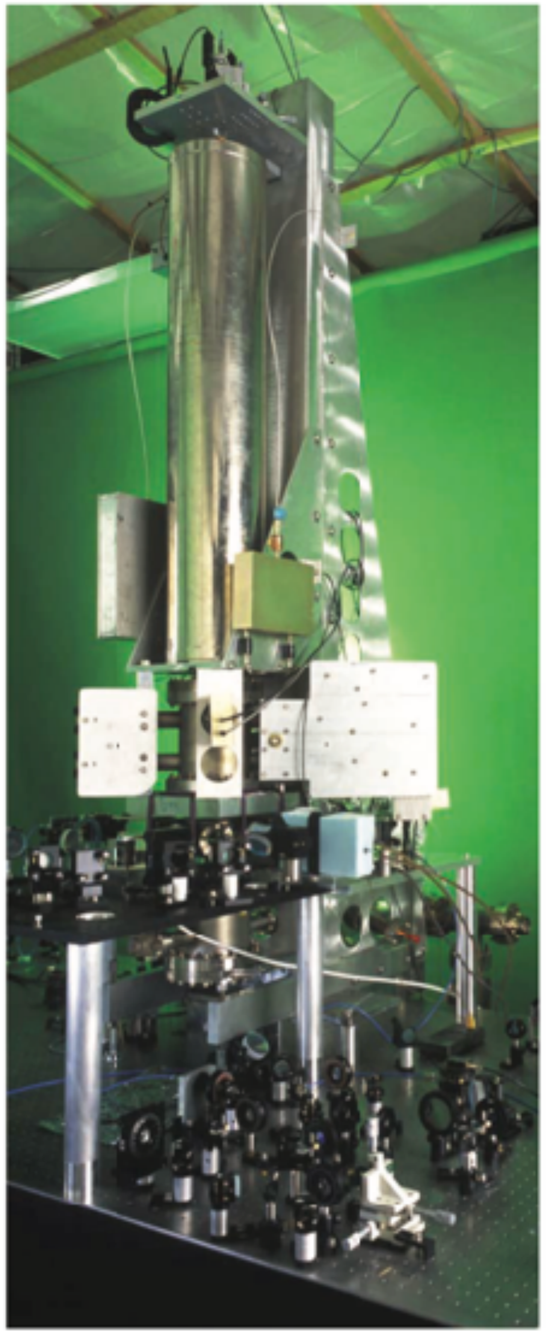
NIST-7, 1993

5×10^{-15}

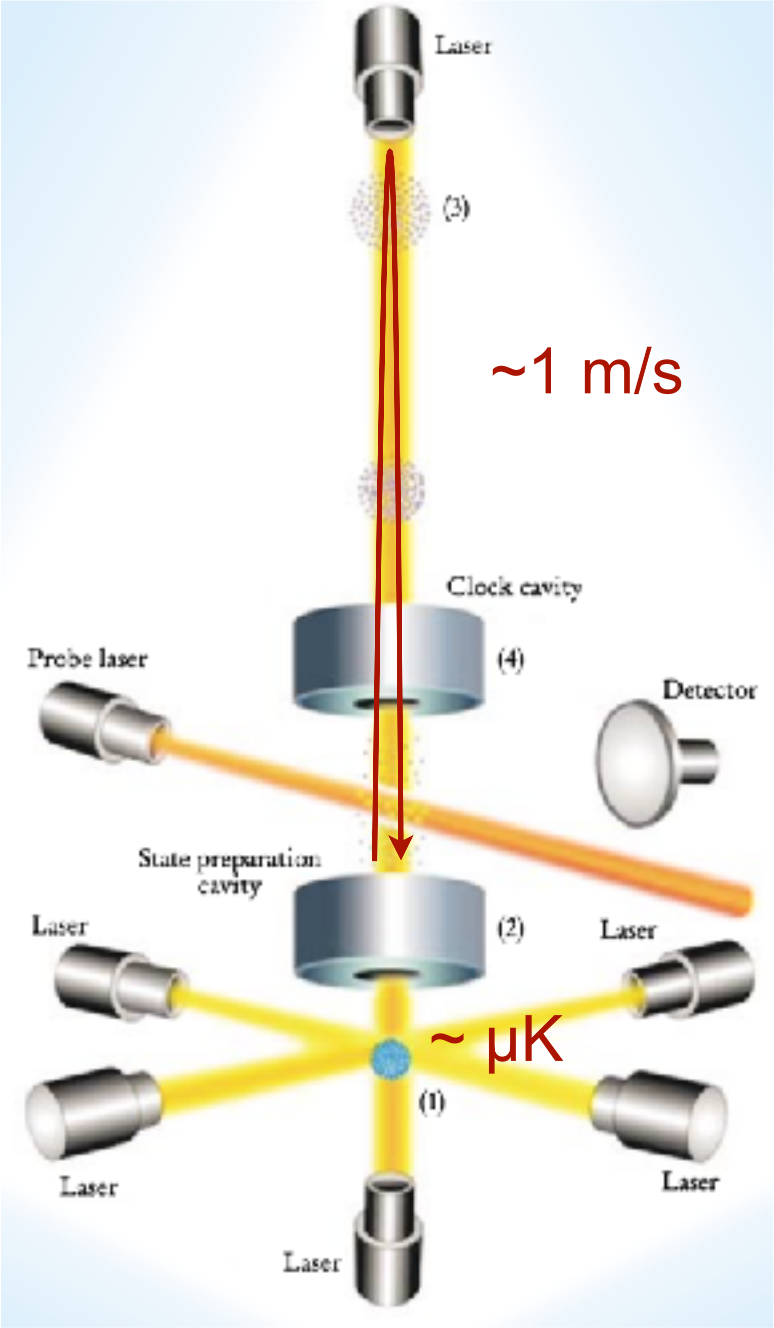




155 cm, 62 Hz



NIST-F1



Cs fountain clock 4×10^{-16}

The Nobel Prize in Physics 1997

Steven Chu

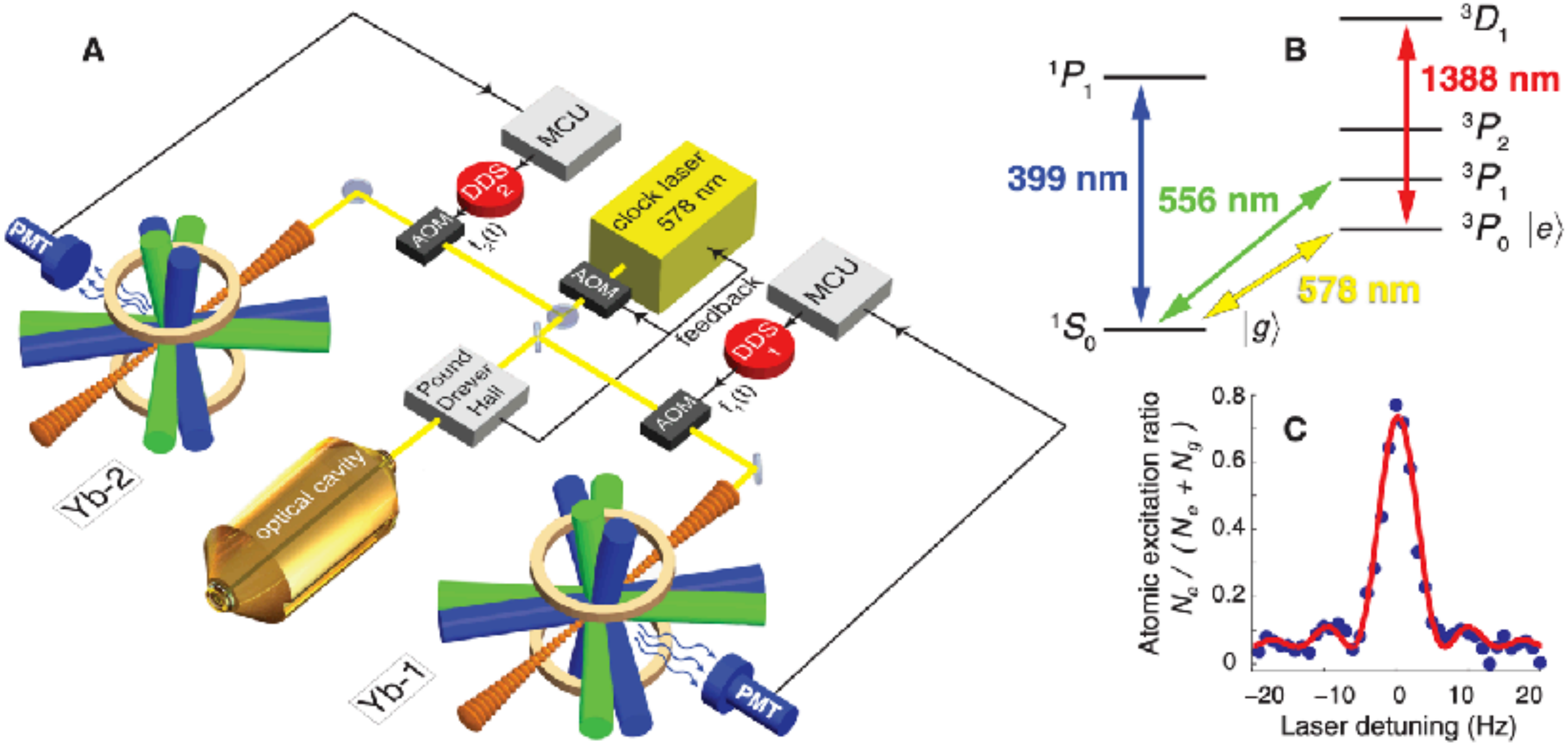
Claude Cohen-Tannoudji

William D. Phillips

"for development of methods to cool and trap atoms with laser light."

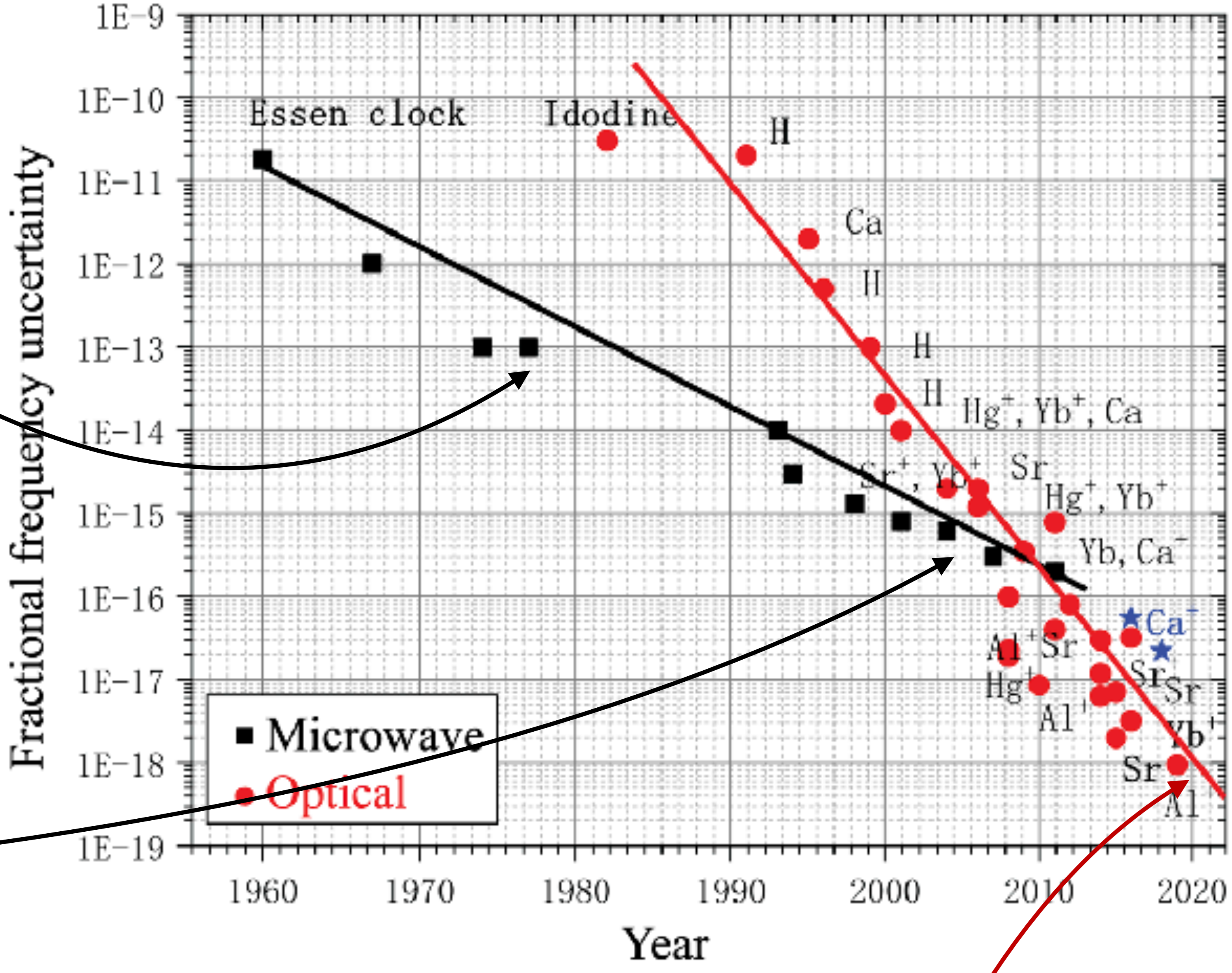
cold atom era !

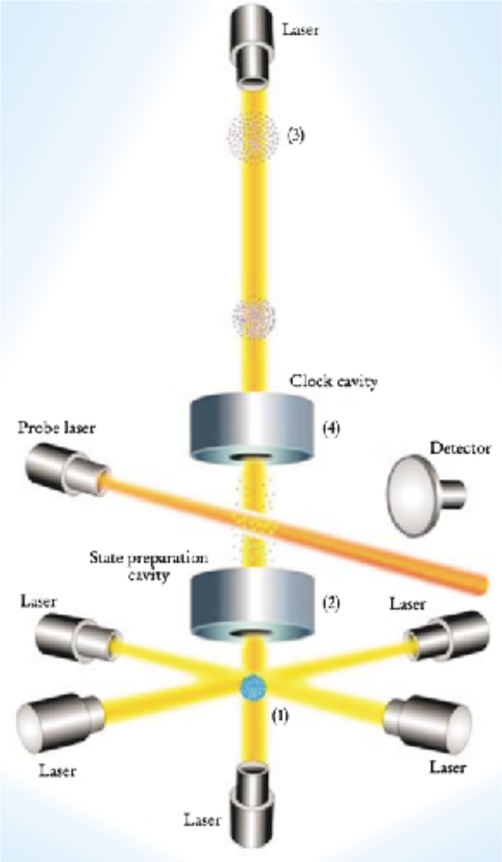
Optical Clock



clock transition frequency ~ 500 THz

Cs atomic clock built for the first GPS satellites (late 1970s)





Cs fountain clock

from **microwave** clock to **optical clock**

the era of 10^{-18}

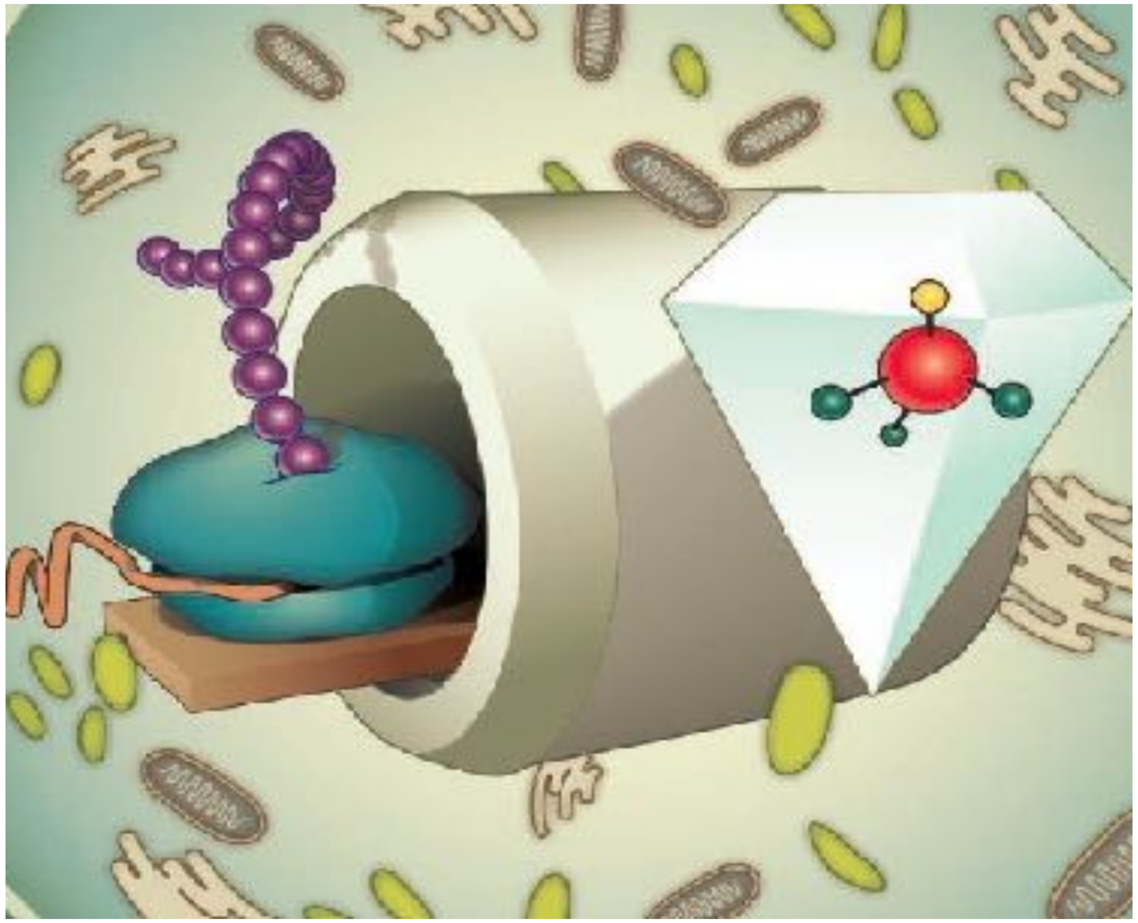
New Features of Quantum Sensing

(1) Spatial Resolution


PHILIPS

Sensing

Toward Molecular-Scale MRI



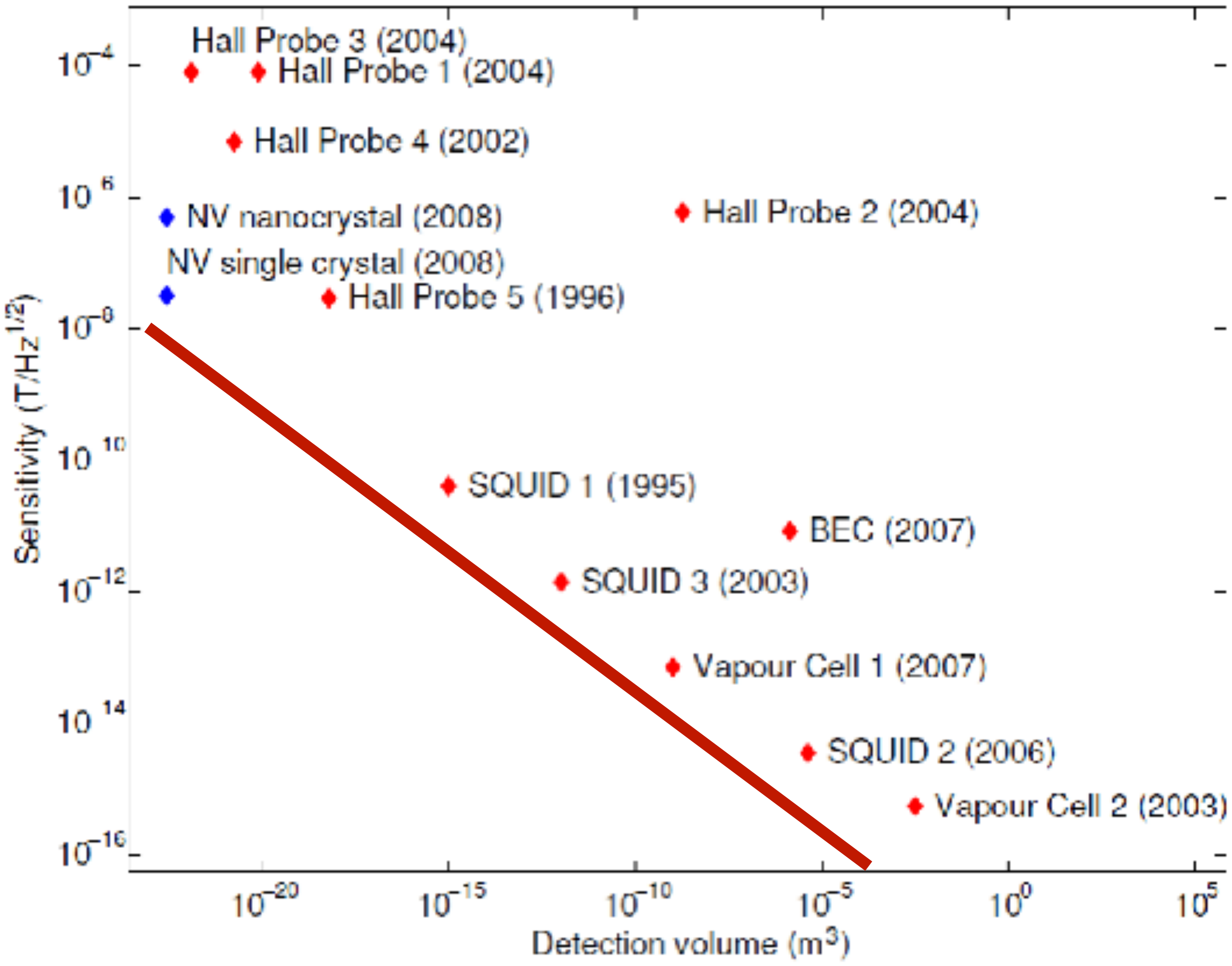
“molecular diagnosis” from *Science* 2013

Sensing single nuclear spins

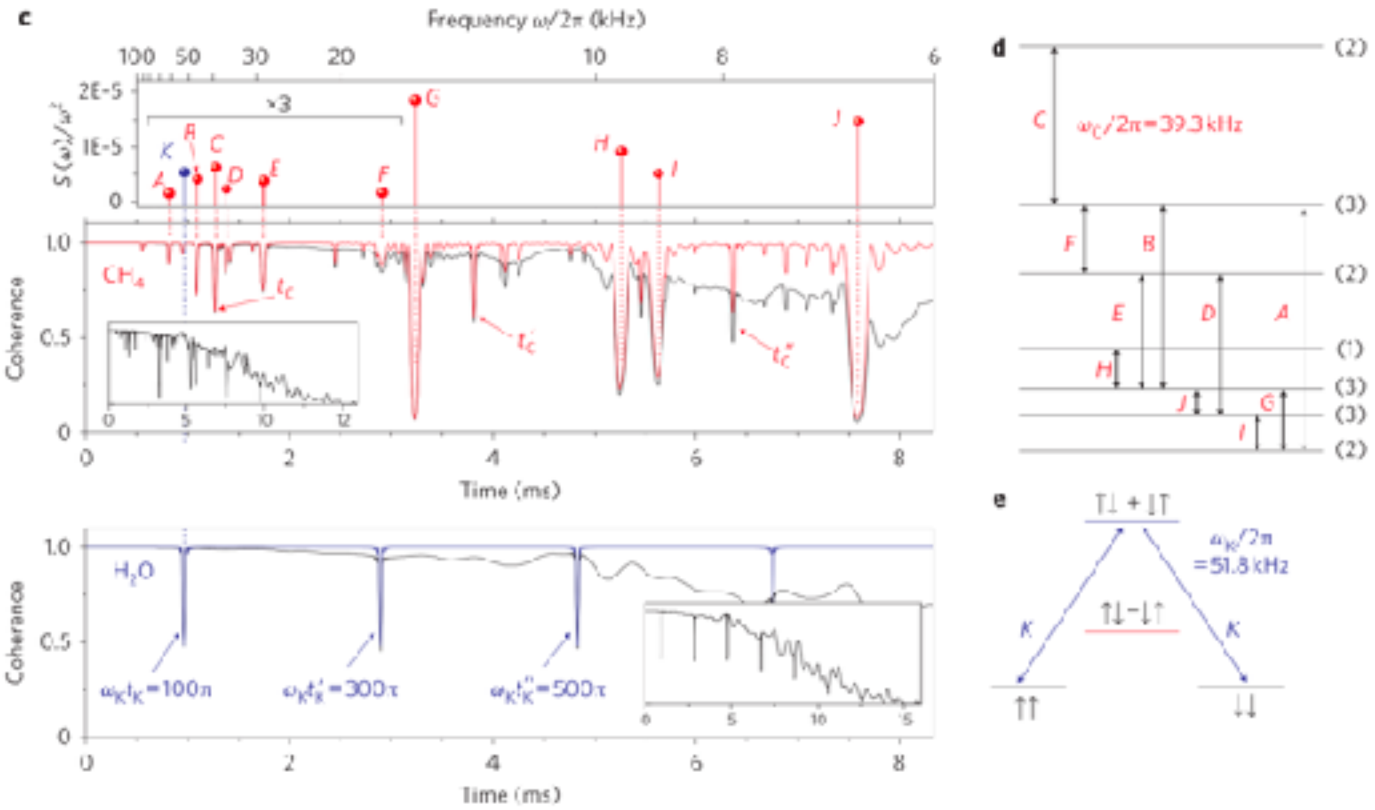
CSRC

Sensing

Resolution vs. Sensitivity

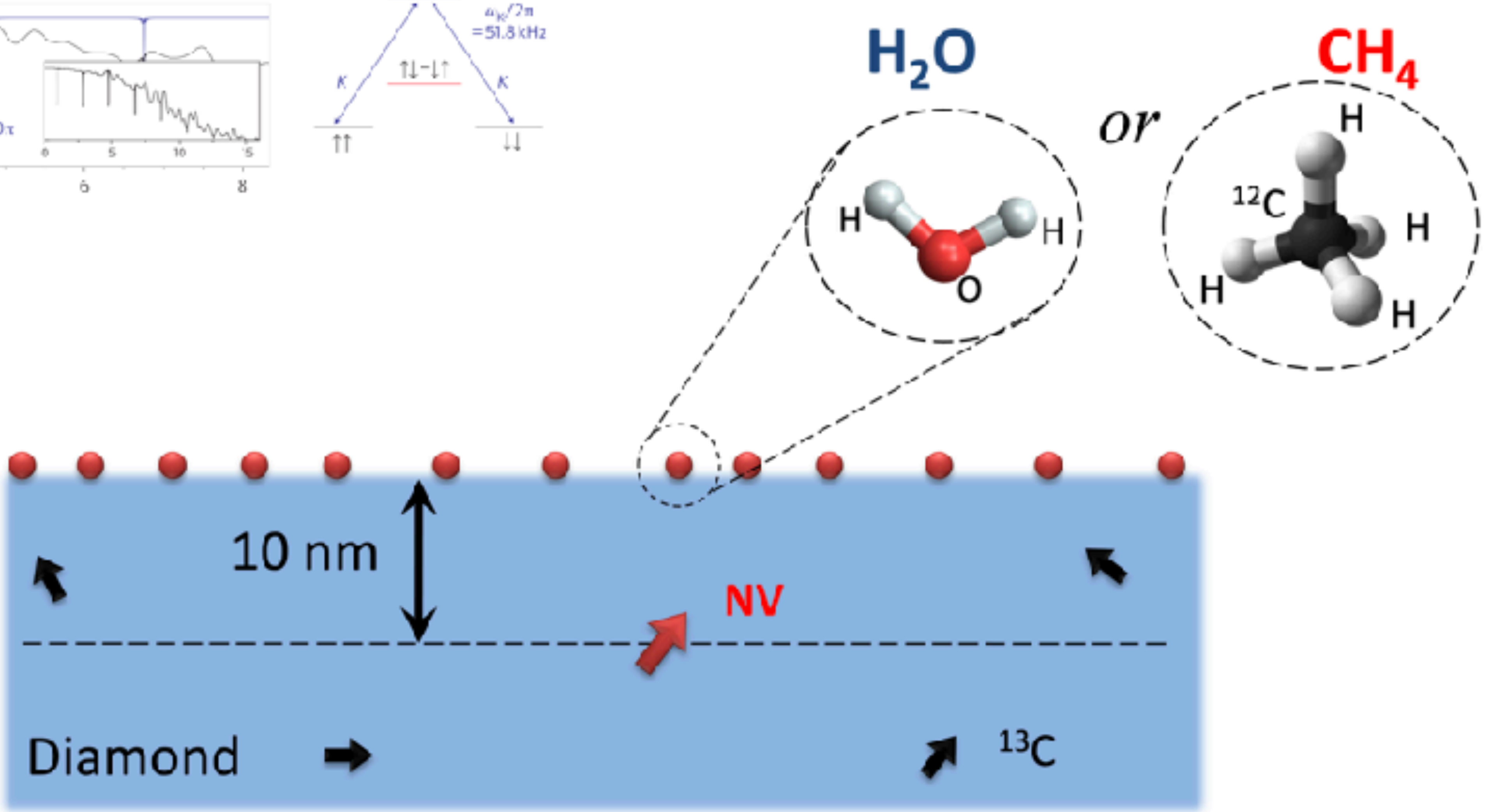


J. R. Maze, et. al. *Nature* (2008)



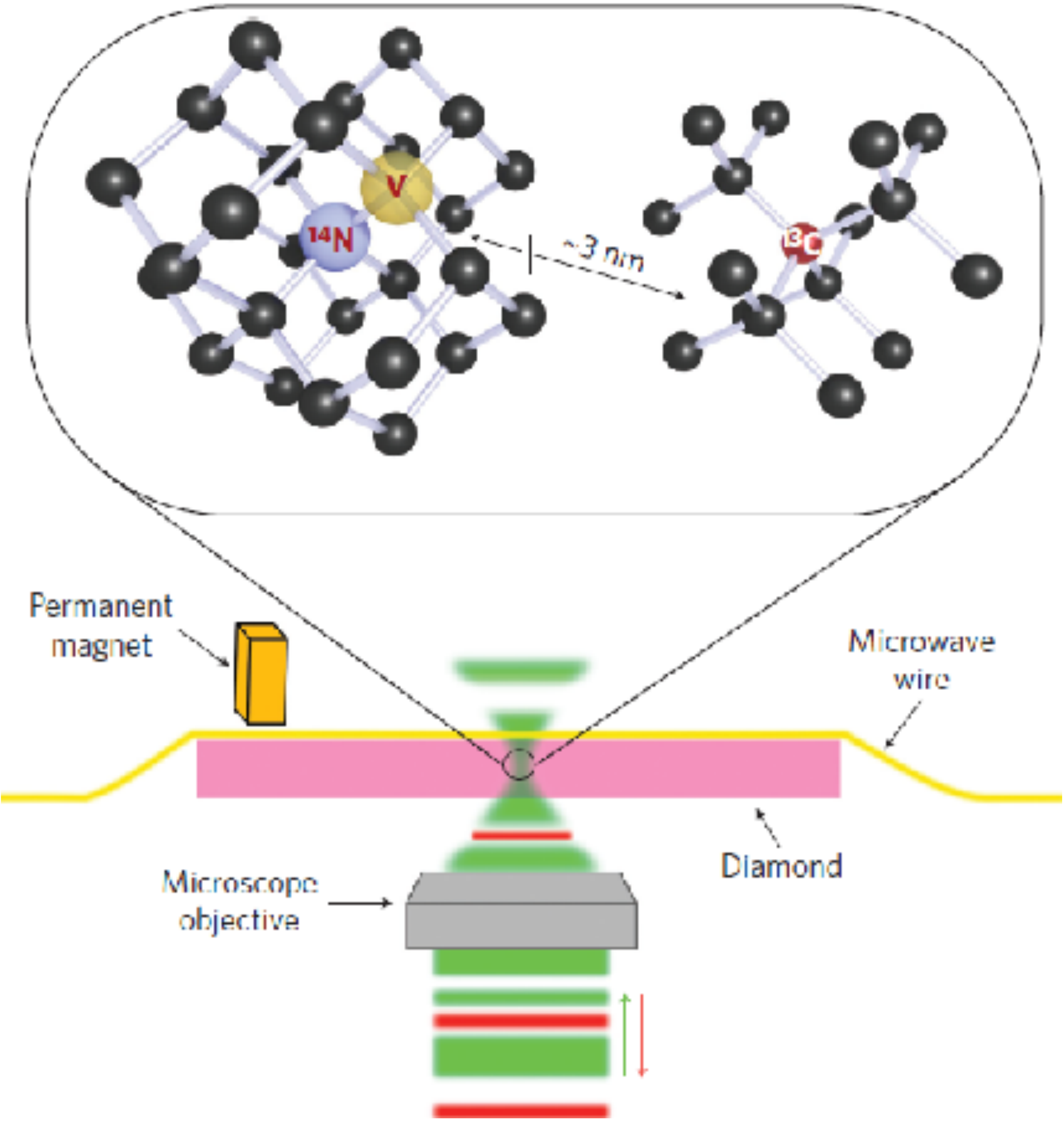
Small Molecules

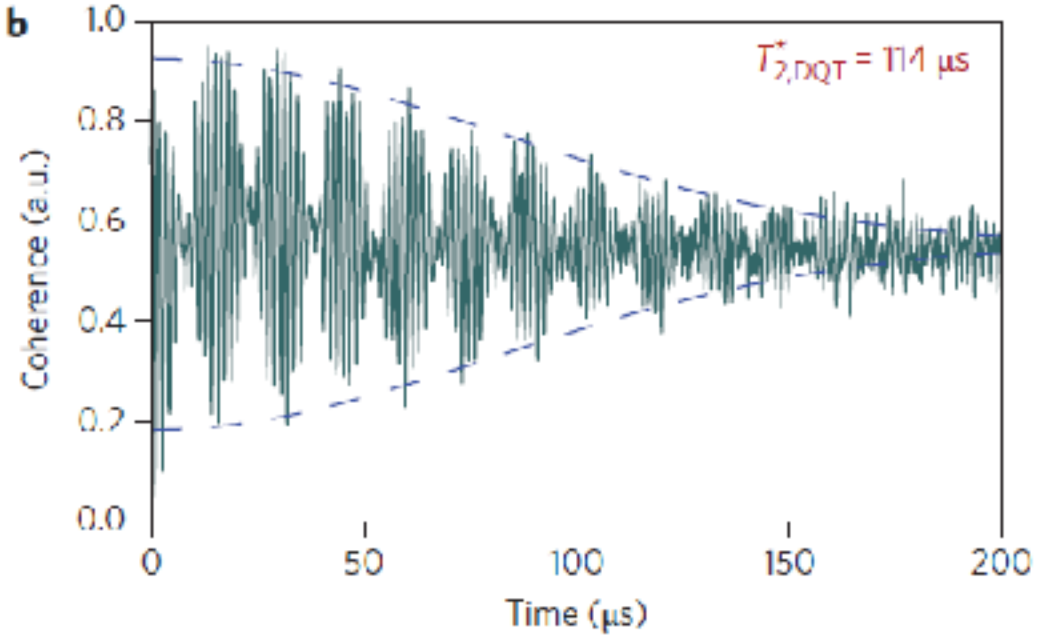
(5 molecules on top of NV are enough)



Sensing

Diamond sample





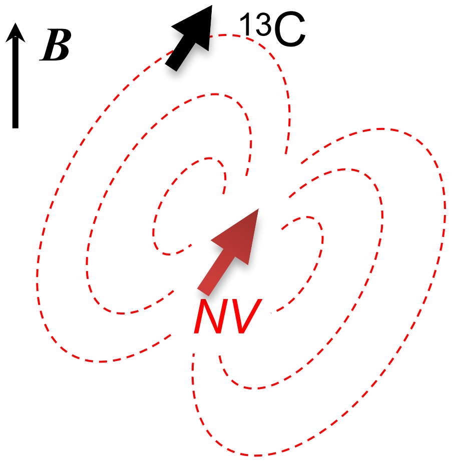
Isotope purified diamond sample:

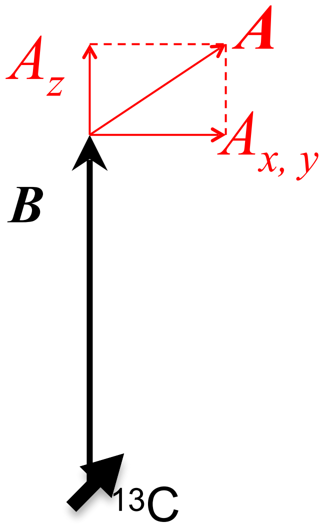
- ^{12}C > 99.99%, 100 times purer than natural abundance
- Typical FID T_2^* time ~ 200 μs
- Typical nearest ^{13}C-NV distance: ~ 3 nm
- Hyperfine coupling strength ~ kHz

Sensing

Microscopic Model

$$H = \Delta S_z^2 - \gamma_e B S_z + S_z \cdot \mathbf{A} \cdot \mathbf{I} - \gamma_C B I_z$$





Conditional Hamiltonian of ^{13}C

$$H_{\mathrm{C13}}(m_S = 0) = -\gamma_C B I_z$$
$$H_{\mathrm{C13}}(m_S = \pm 1) = -\gamma_C B I_z \pm \mathbf{A} \cdot \mathbf{I}$$

Effective Larmor frequency for given m_S

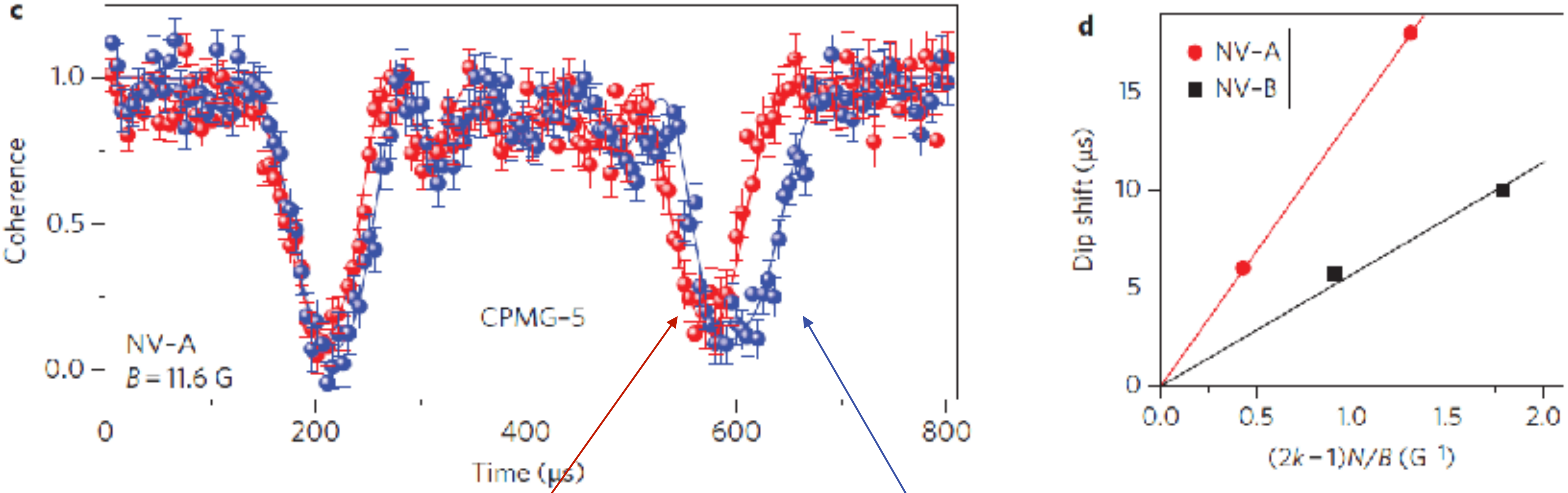
$$2\pi f_{\mathrm{Larmor}}(m_S) = -\gamma_C B + m_S \cdot A_z$$

$$\text{for} \quad |\gamma_C B| \gg |\mathbf{A}|$$

Sensing

Effective Larmor frequency for given transition

$$2\pi f_{\text{Larmor}}(0 \leftrightarrow m_S) = -\gamma_C B + m_S \cdot A_z/2$$

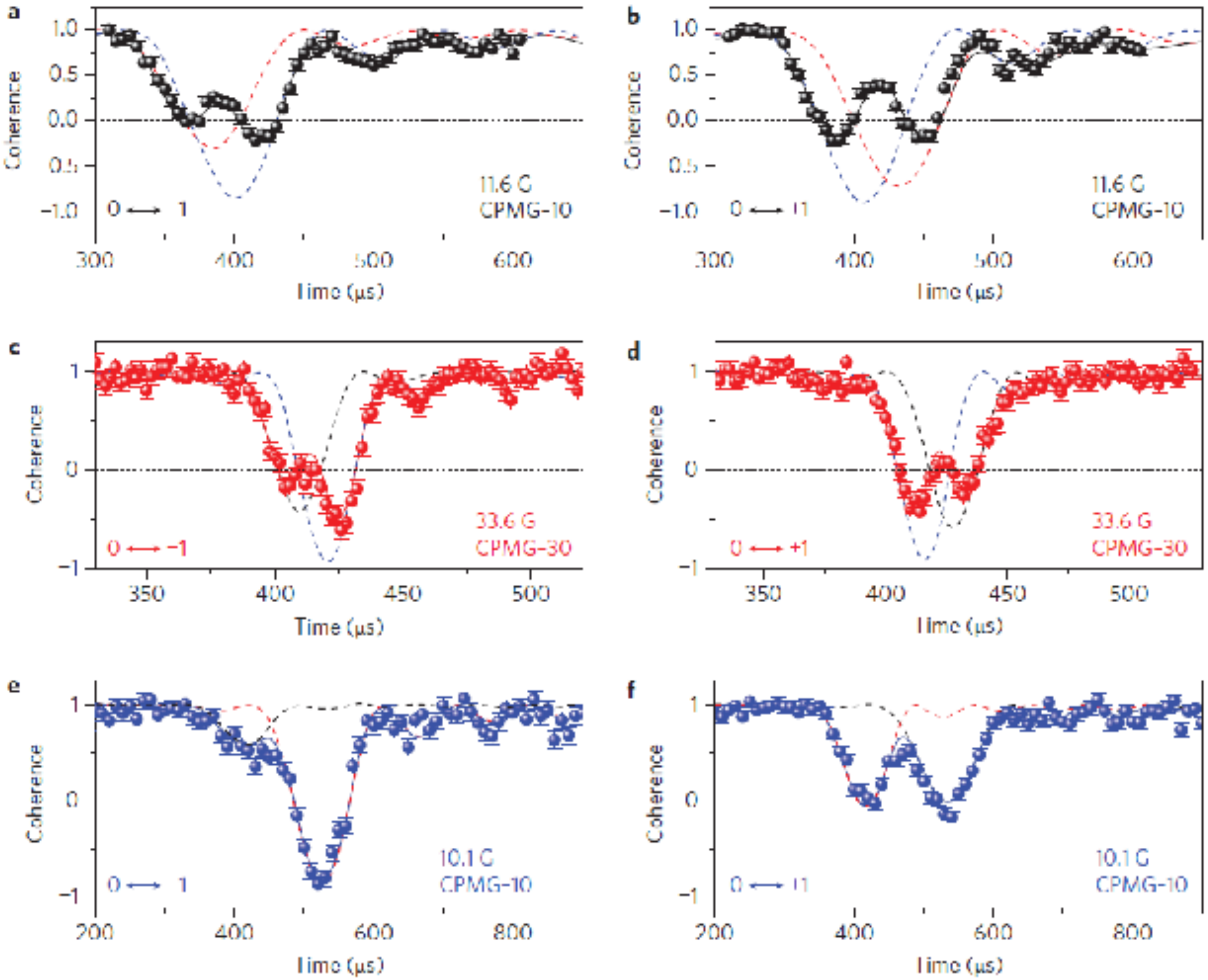


$0 \leftrightarrow +1$

$0 \leftrightarrow -1$

Sensing

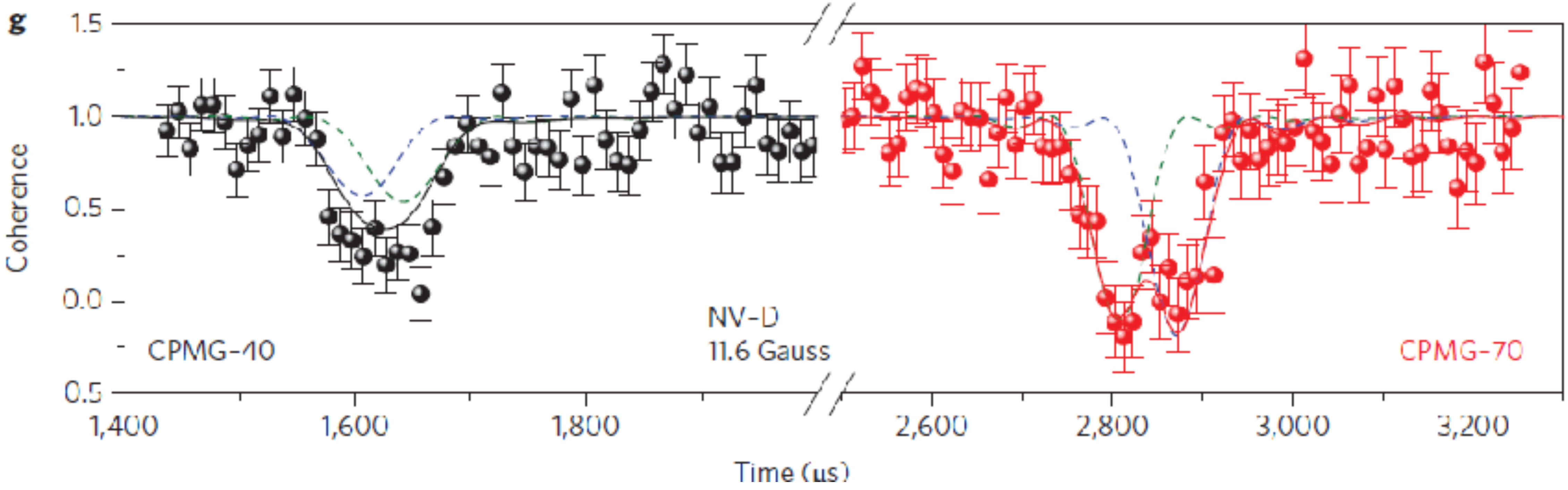
More than one ^{13}C nuclear spins



Positions of each ^{13}C nuclear spins could be identified through the ***B*** fields dependence

Sensing

Proof-of-Principle measurement for single nuclear spin NMR



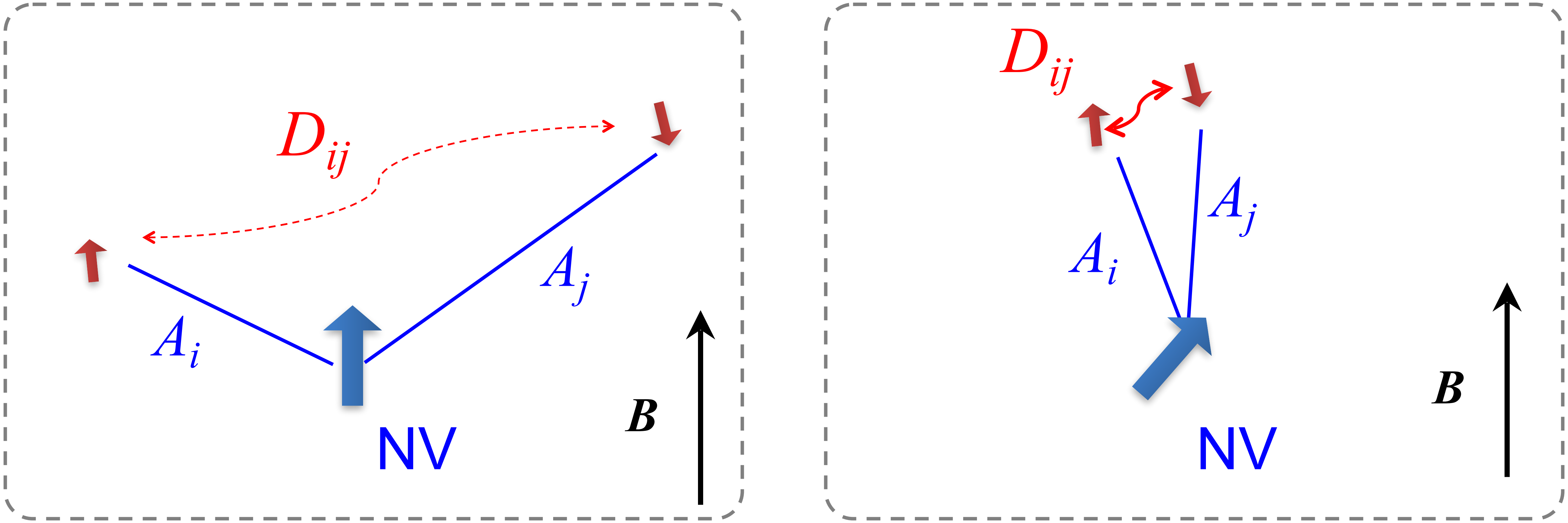
long coherence time T_2 ~ 3 ms

measured hyperfine coupling ~300 Hz
distance: ~ 3 nm from NV center

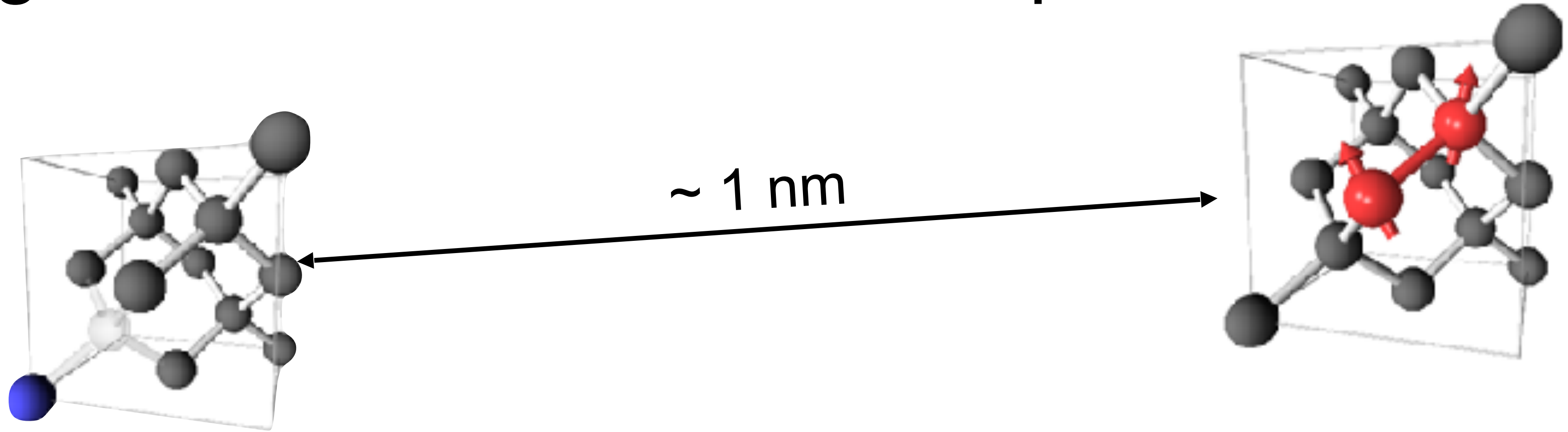
CSRC

Sensing

From non-interacting nuclear spins to interacting clusters



Unraveling interaction within nuclear spin clusters in diamond



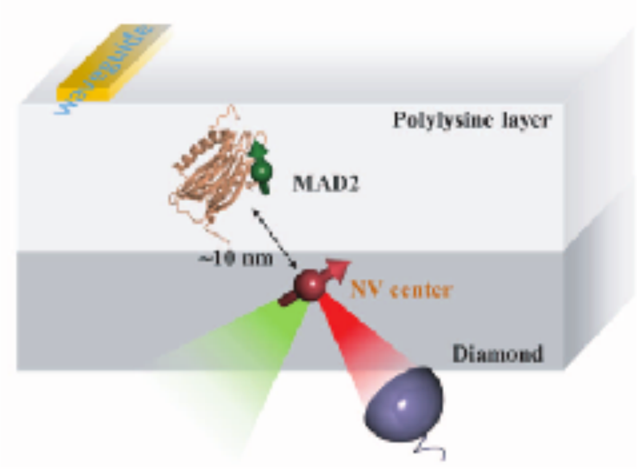
Theory: Zhao, N., et. al, *Nat. Nanotech.* **6**, 242 (2011)
Experiment: Shi, F.Z., et. al, *Nat. Phys*. **10**, 21 (2014)

Sensing

Nano-scale NMR/ESR/MRI with NV

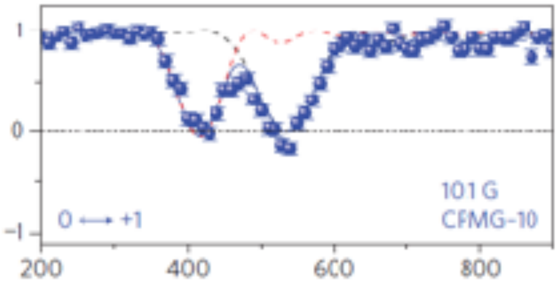
sensing electron spins

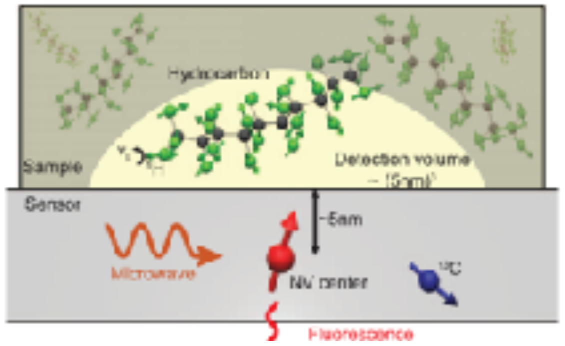
1. Grinolds, M. S., et. al, *Nat. Phys.* **9**, 215 (2013) [Harvard]
2. Sushkov, O., et. al, *Nano Lett.* **14**, 6443 (2014) [Harvard]
3. Shi, F.Z., et. al, *Science* **347**, 1135 (2015) [USTC]



sensing nuclear spins

4. Zhao, N., et. al, *Nat. Nanotech.* **7**, 657 (2012) [Stuttgart]
5. Kolkowitz, S., et. al, *PRL* **109**, 137601 (2012) [Harvard]
6. Taminiau, T. H., et. al, *PRL* **109**, 137602 (2012) [Delft]
7. Staudacher, T., et. al, *Science* **339**, 561 (2013) [Stuttgart]
8. Müller, C., et. al, *Nat. Comm.* **5**, 4703 (2014) [Ulm]
9. Zhao, N., et. al, *Nat. Nanotech.* **6**, 242 (2011) [HK, theory]
10. Shi, F.Z., et. al, *Nat. Phys.* **10**, 21 (2014) [USTC, exp.]





The Next Breakthrough

single-proton & single-molecule outside diamond

New Features of Quantum Sensing

(2) System Miniature

CSRC

Navigation Methods

compass

sextant

radio

radar

satellite (GPS)

Observation of the Outside World !

Inertial Navigation System

An **inertial navigation system** (**INS**) is a navigation aid that uses a computer, motion sensors (**accelerometers**) and rotation sensors (**gyroscopes**) to continuously calculate via dead reckoning the position, orientation, and velocity (direction and speed of movement) of a moving object **without the need for external references**.

Inertial Navigation System

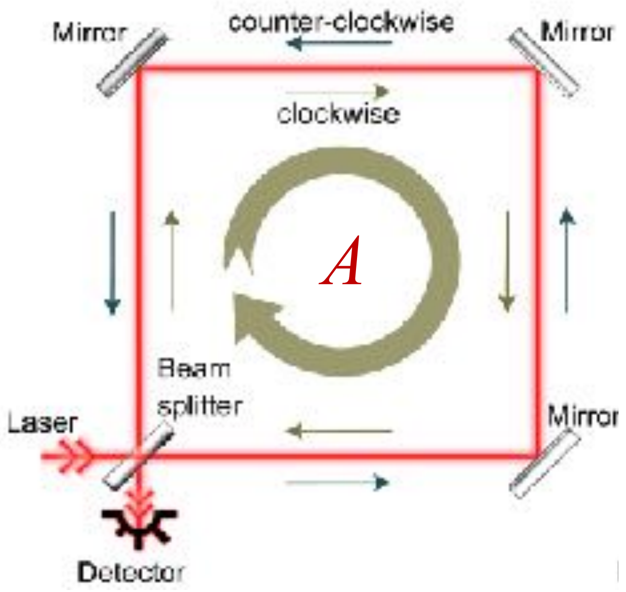
gyroscope: an example

Classical Gyroscope

级别	陀螺种类	惯导方式	角度随机游走° /√h	漂移(° /h)	尺寸	价格	主要技术瓶颈	主要应用场景
战略级	静电陀螺	平台	—	$10^{-4} \sim 10^{-6}$	大	高	悬浮球加工工艺和稳定性要求高,控制系统复杂	核潜艇
	液浮陀螺 三浮陀螺	平台	—	$10^{-3} \sim 10^{-5}$	大	高	结构偏心,转轴摩擦	潜艇,舰船,远程火箭
战术级	挠性陀螺 动力调谐陀螺	可平台也可捷联	—	$10^{-2} \sim 10^{-3}$	较小	较低	调谐参数的精确设定,结构热和应力稳定性	飞机,火箭,导弹
	激光陀螺	捷联	$10^{-3} \sim 10^{-4}$	10^{-2}(无温控时>10^{-2}) ~ 10^{-4}(有温控时可用于核潜艇)	中等	中	闭锁效应,光的散粒噪声	飞机,火箭,导弹,卫星
	光纤陀螺	捷联	$10^{-1} \sim 10^{-4}$	$10^{-3} \sim 10^{-4}$	中等	中	光源频率稳定性,光纤热稳定性	飞机,火箭,导弹,卫星
消费级	振动陀螺 MEMS陀螺	捷联	$10^{0} \sim 10^{-2}$	10^{1}(国内) ~ 10^{-1}(国外)	小	较低	加工相对精度,等效热噪声	常规弹药制导化,消费电子产品

Ring Laser Gyroscope

Sagnac effect



$$\Delta\phi = \frac{8\pi A}{\lambda c}\omega$$

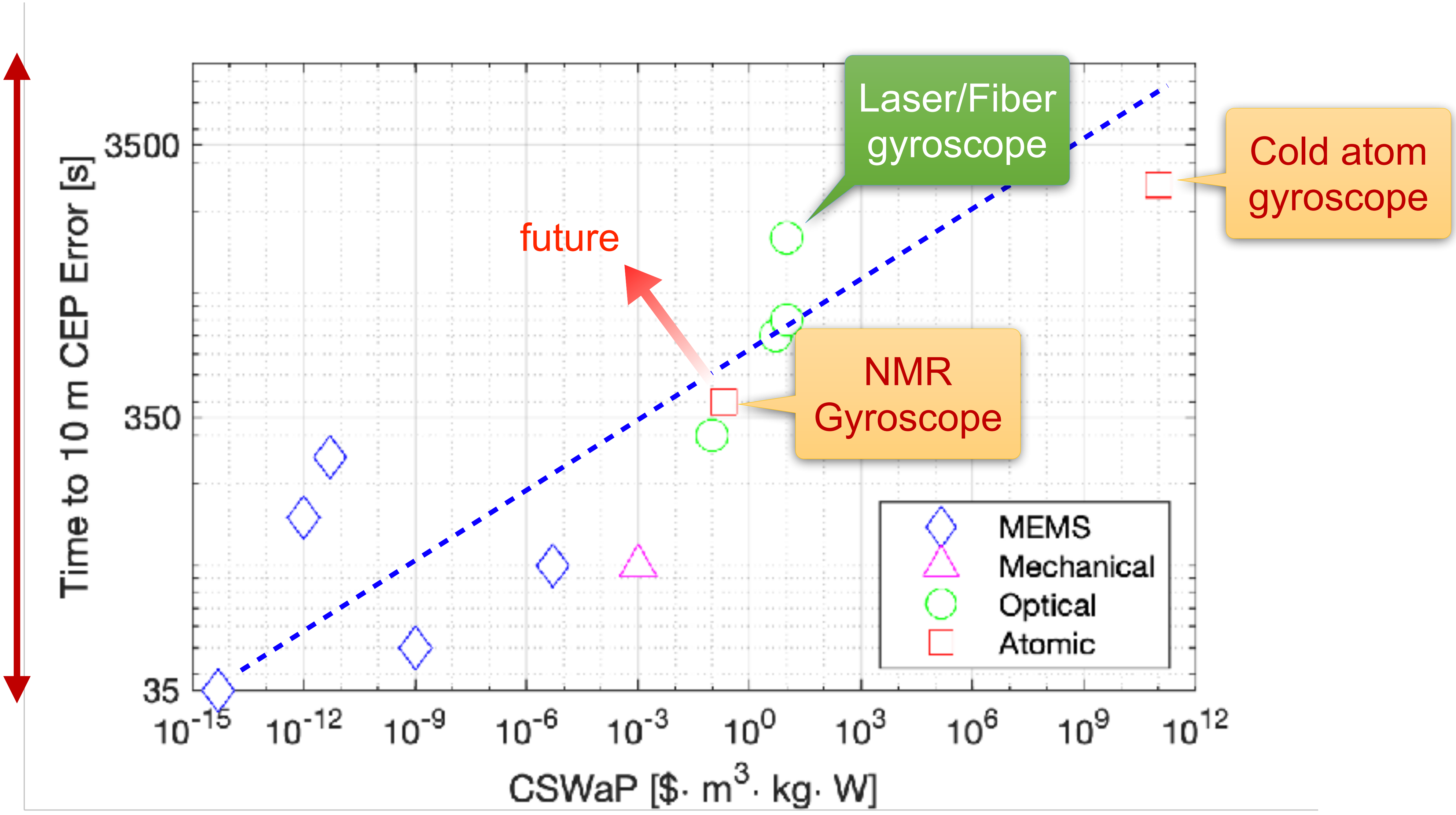
size: ~ 1000 cm^3

weight: ~ kg

Gyroscope Performance

high precision

low precision



Laser/Fiber gyroscope

Cold atom gyroscope

future

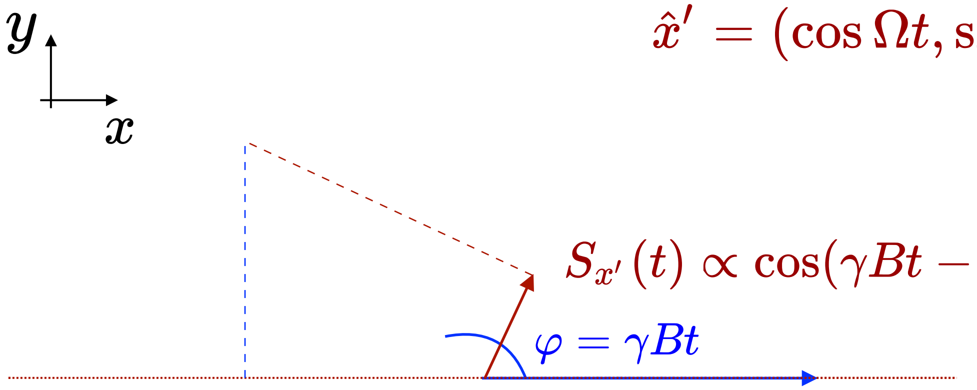
NMR Gyroscope

small, portable, cheap, …

big, heavy, expensive…

Quantum Gyroscope: Spin-Based Systems

Spin precession in static and rotating frame



$\hat{x}' = (\cos \Omega t, \sin \Omega t)$

$S_{x'}(t) \propto \cos(\gamma B t - \Omega t)$

$\varphi = \gamma B t$

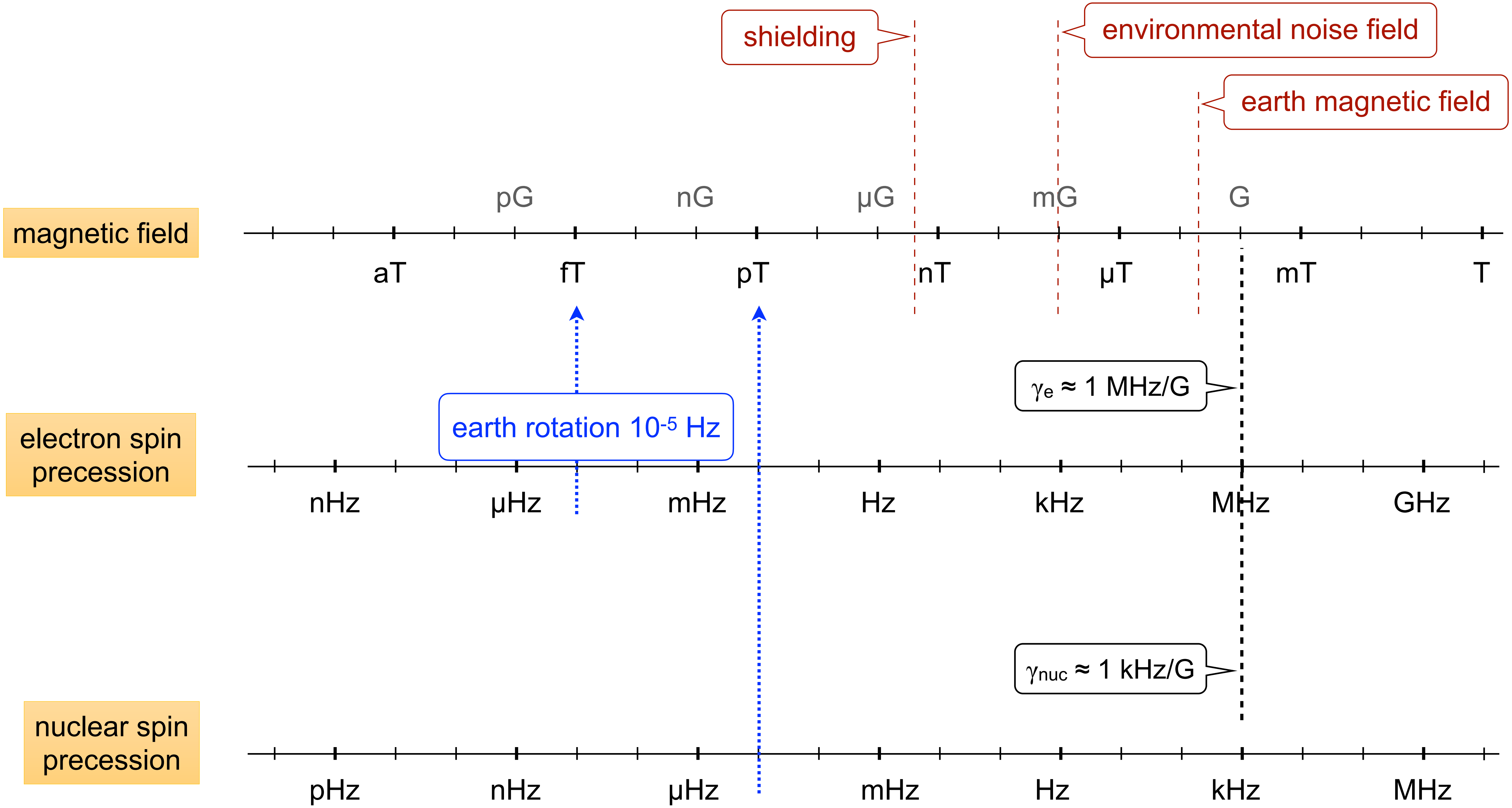
$S_x(t) \propto \cos(\gamma B t)$

in a rotating frame with angular velocity Ω

$$\gamma B t - \Omega t \equiv \gamma B_{\text{meas}} t$$

Gyroscope = Magnetometer

$$B_{\text{gyro}} = B - B_{\text{meas}} = \Omega / \gamma$$



shielding

environmental noise field

earth magnetic field

magnetic field

pG nG µG mG G

aT fT pT nT µT mT T

$\gamma_e \approx 1$ MHz/G

earth rotation 10^{-5} Hz

electron spin precession

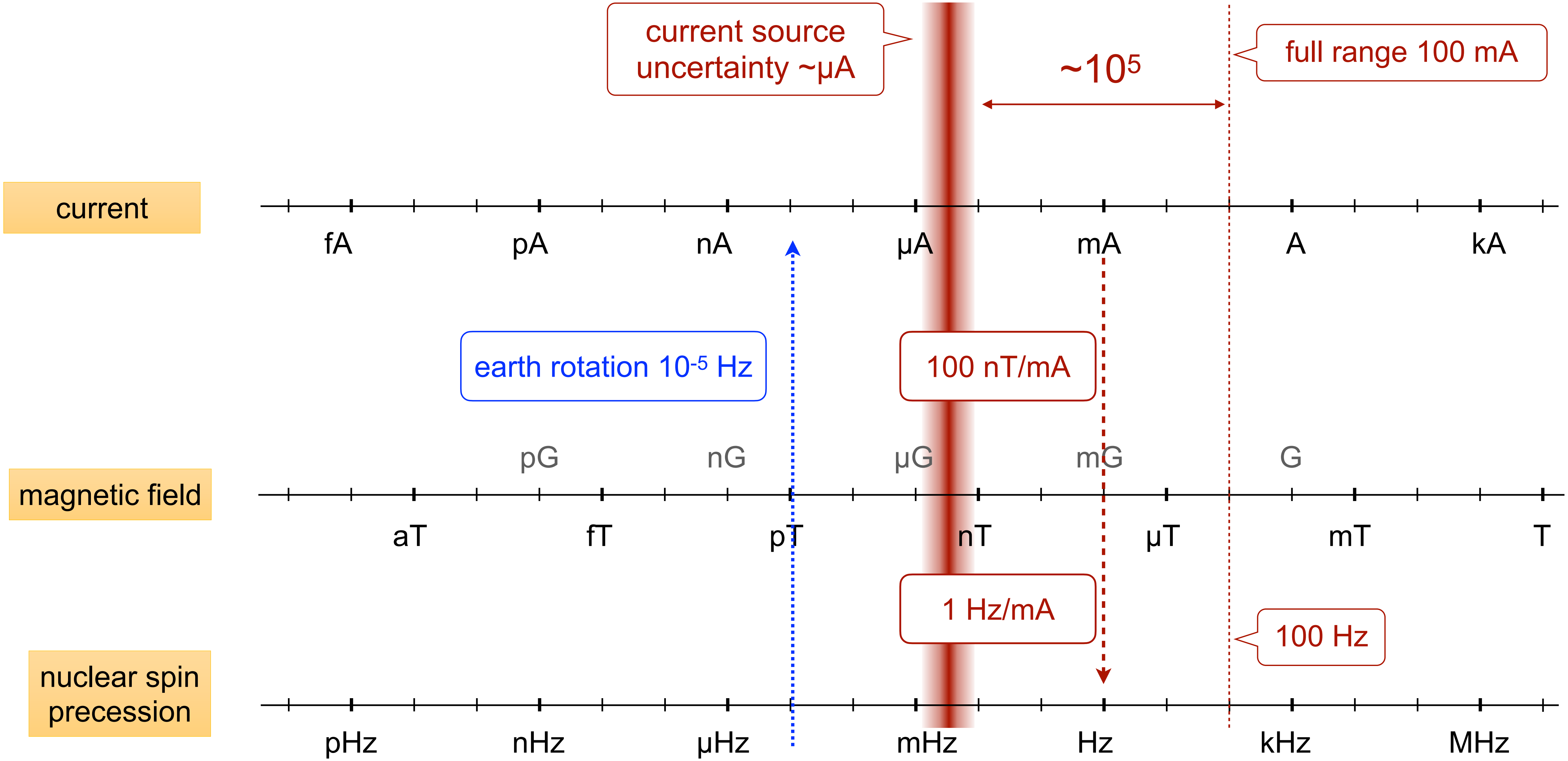
nHz µHz mHz Hz kHz MHz GHz

$\gamma_{nuc} \approx 1$ kHz/G

nuclear spin precession

pHz nHz µHz mHz Hz kHz MHz

$$360^\circ/\mathrm{day} = 15^\circ/\mathrm{h} \approx 10^{-5}\mathrm{Hz}$$



current

magnetic field

nuclear spin precession

current source uncertainty ~μA

~10^5

full range 100 mA

earth rotation 10^{-5} Hz

100 nT/mA

1 Hz/mA

100 Hz

$$\gamma B t - \Omega t \equiv \gamma B_{\text{meas}} t$$

Two Species: ^{129}Xe & ^{131}Xe

$$\gamma_{129} = -2\pi \times 11.86\,\mathrm{mHz/nT}$$

$$\gamma_{131} = 2\pi \times 3.516\,\mathrm{mHz/nT}$$

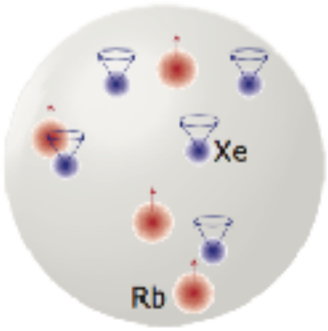
$$\Omega_{129} = \gamma_{129} B - \Omega$$

$$\Omega_{131} = \gamma_{131} B - \Omega$$

$$\boxed{\Omega = \frac{\gamma_{129}\Omega_{131} - \gamma_{131}\Omega_{129}}{\gamma_{131} - \gamma_{129}}}$$

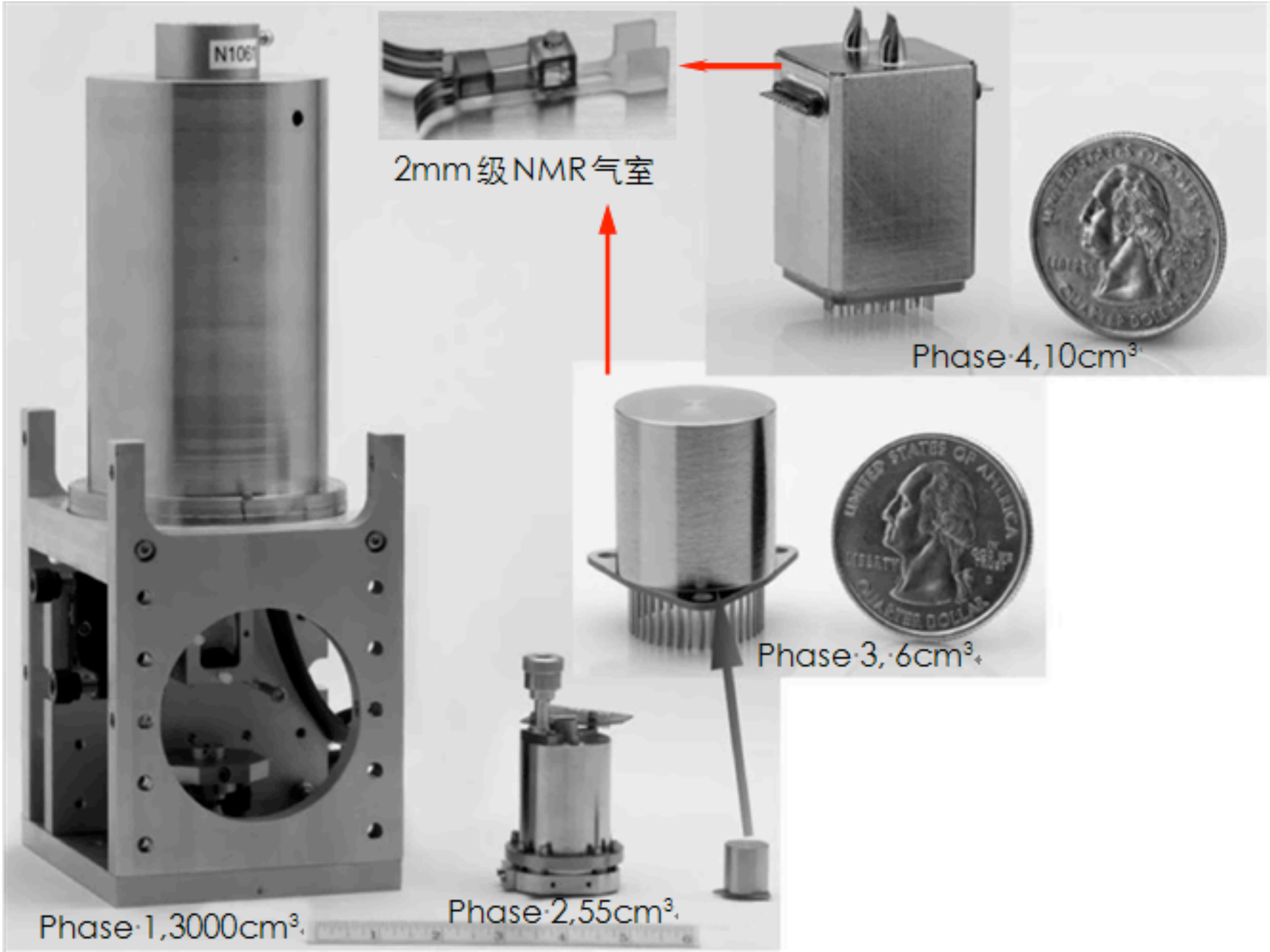
magnetic fluctuations (due to current noise) can be eliminated.

NMR Measurement



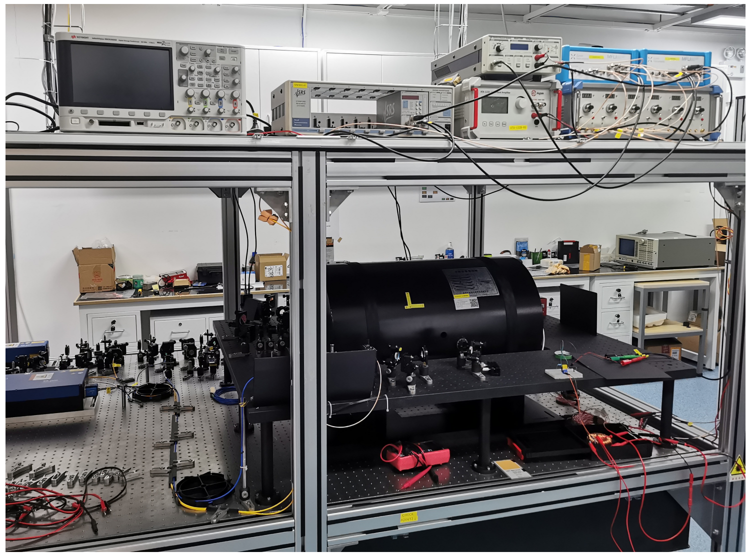
$$\begin{tikzcd}
\text{unpolarized atoms} \arrow[r, "\text{optical pumping}"] & \langle S_z \rangle \arrow[r, "\text{exchange pumping}"] & \langle I_z \rangle \arrow[dll, "\text{resonant driving}"'] \\
\langle I_y \rangle (t) \arrow[r, "\text{precession}"] & \langle S_x \rangle (t) \arrow[r, "\text{Faraday rotation}"] & \text{NMR signal}
\end{tikzcd}$$

Micro-NMR Gyroscope by Northrop Grumman Corporation

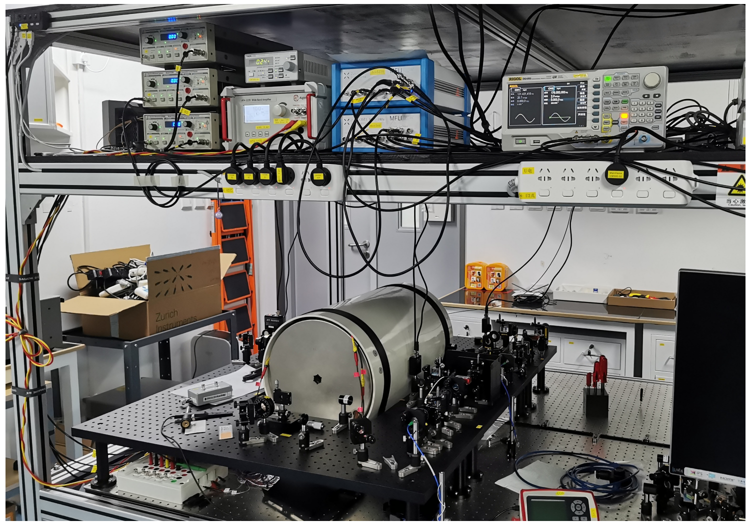


实物“产品”

物理平台

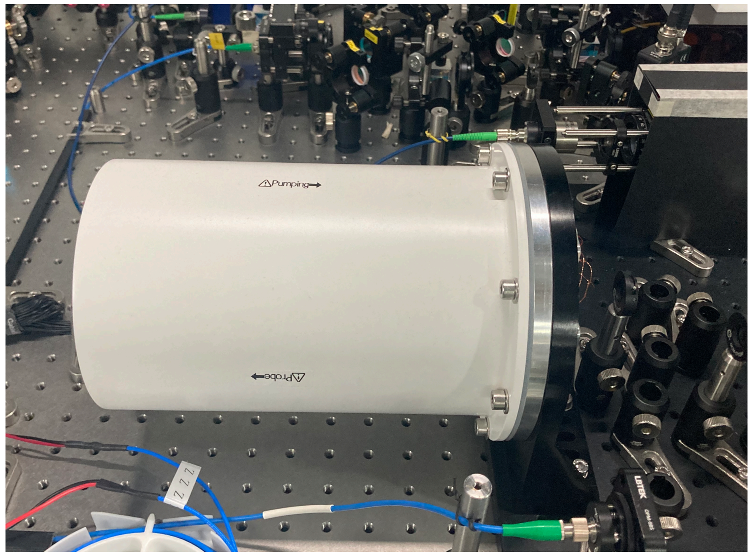


测试系统 #1

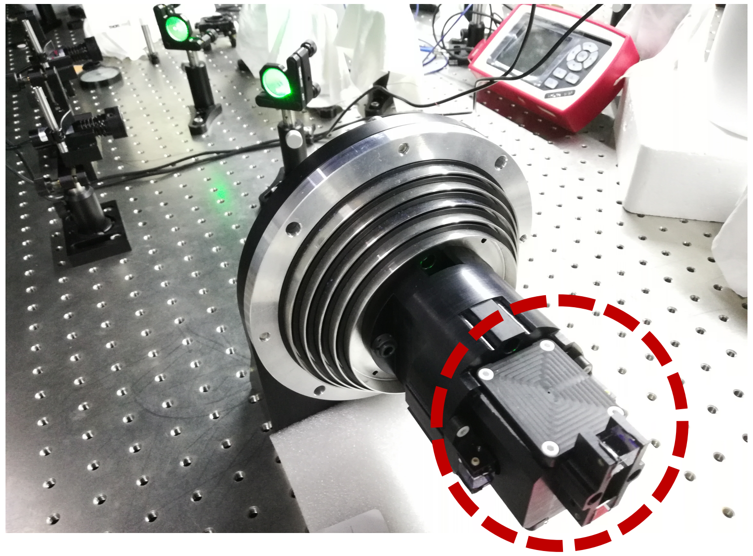


测试系统 #2

小型化样机*



3000cm^{3}样机（外观）



3000cm^{3}样机（内部）

* 设计加工：总体工程所　　调试测试：计科中心

Quantum Sensing

1. What ?
2. Why ?
3. How ?

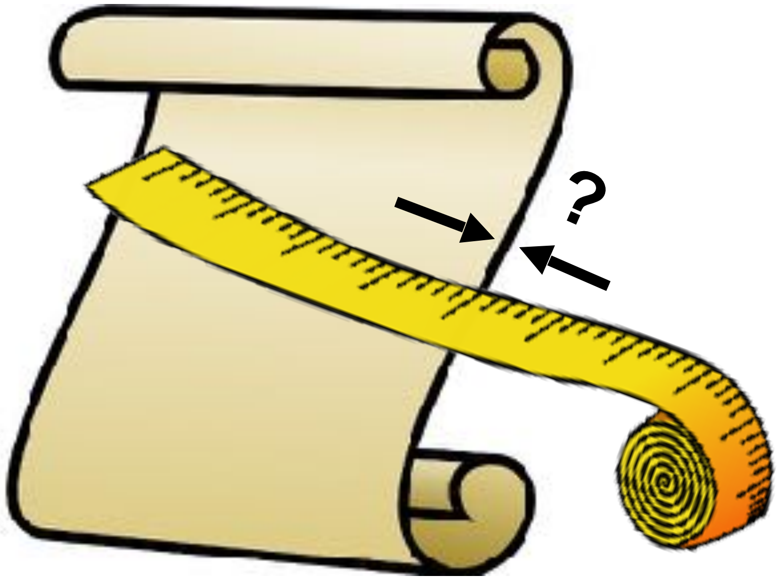
CSRC

Quantum Sensor is a toolbox

clock, magnetometer, gyroscope, interferometer, electric field sensor, gravitational wave sensor…

thermal atoms, cold atoms, atomic spins, solid-state defects, lasers, superconducting devices, …

How to measure the thickness of a piece of paper?



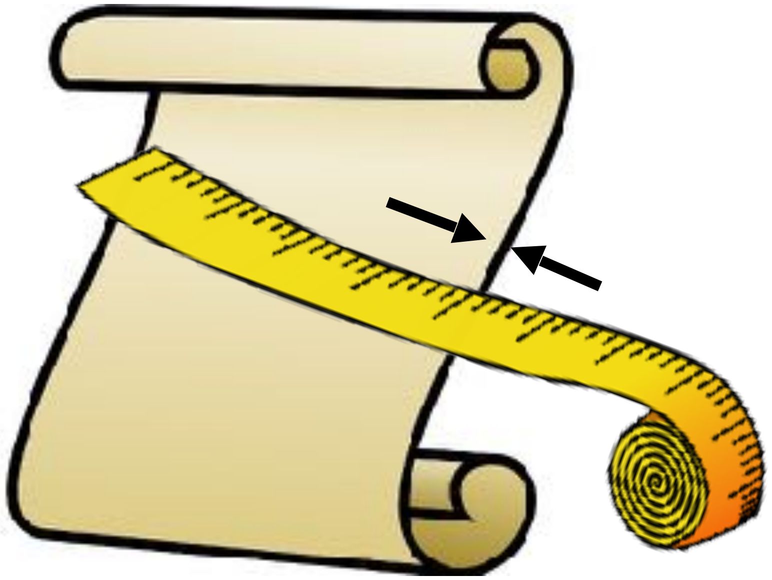
what kind of **equipment** do you need?

what kind of **“resource”** do you need?

uncertainty:	> mm	1~0.1 mm	~0.01 mm	~0.001 mm

How to measure…

thickness of a piece of paper ?



or
more clever

single meas.:	$d \pm \Delta_d$	$D \pm \Delta_d$
multiple meas.:	$\bar{d} \pm \Delta_d/\sqrt{N}$	$D/N \pm \Delta_d/N$

$\sqrt{N}$ enhancement

- ruler uncertainty
  - signal-to-noise ratio (SNR)

- page vs. book
  - system coherence

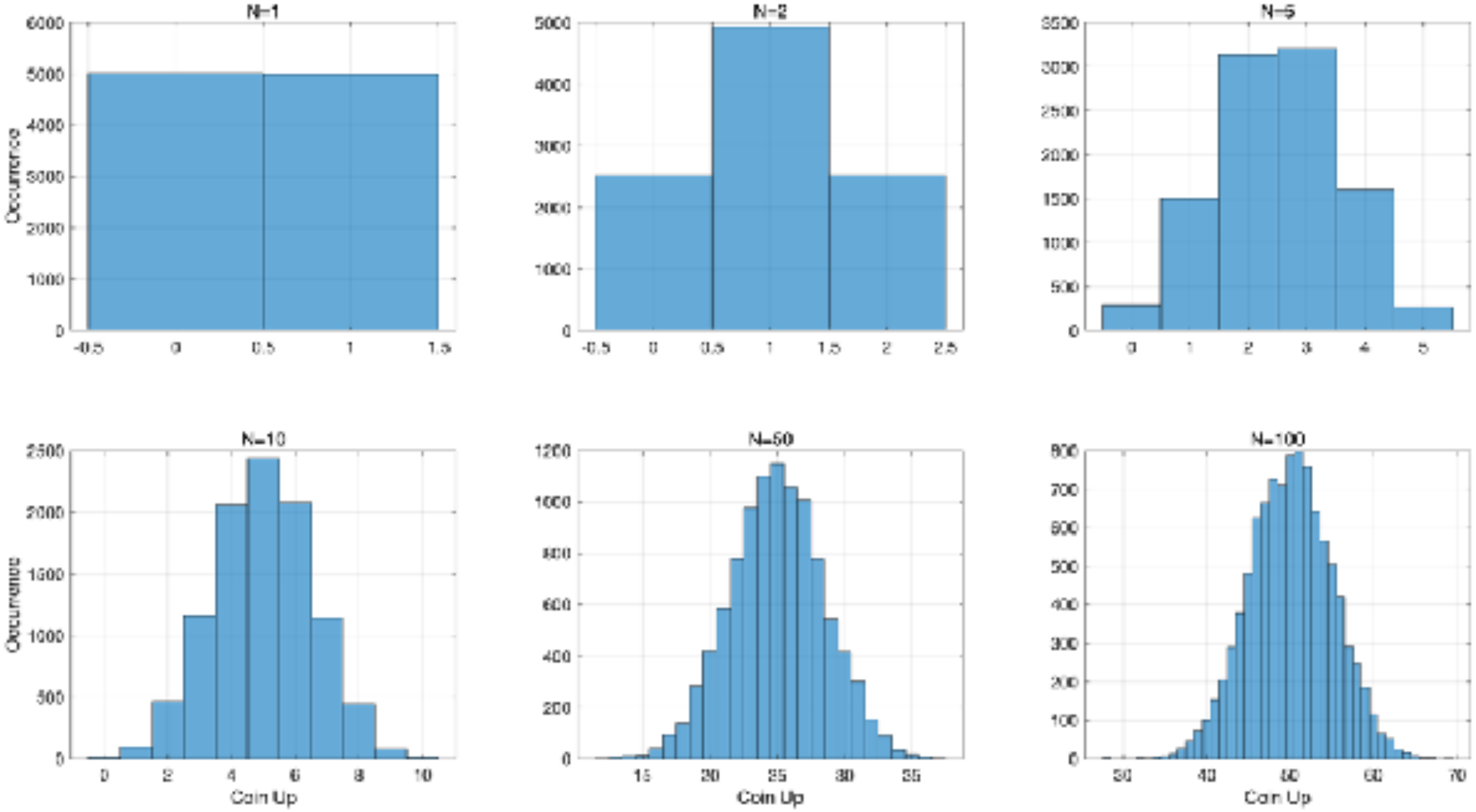
Central Limit Theorem

- independent and identically distributed random variables $\{X_1, X_2, \ldots\}$
- expected value: $\mathbb{E}(X_i) = 0$; variance : $\mathrm{var}(X_i) = \sigma^2 < \infty$

N-sample average $\bar{X}_N = \dfrac{X_1 + X_2 + \cdots + X_N}{N}$ approaches to **normal distribution.**

- expected value: $\mathbb{E}(\bar{X}_N) = 0$; variance : $\mathrm{var}(\bar{X}_N) = \dfrac{\sigma^2}{N}$

standard deviation
(uncertainty)

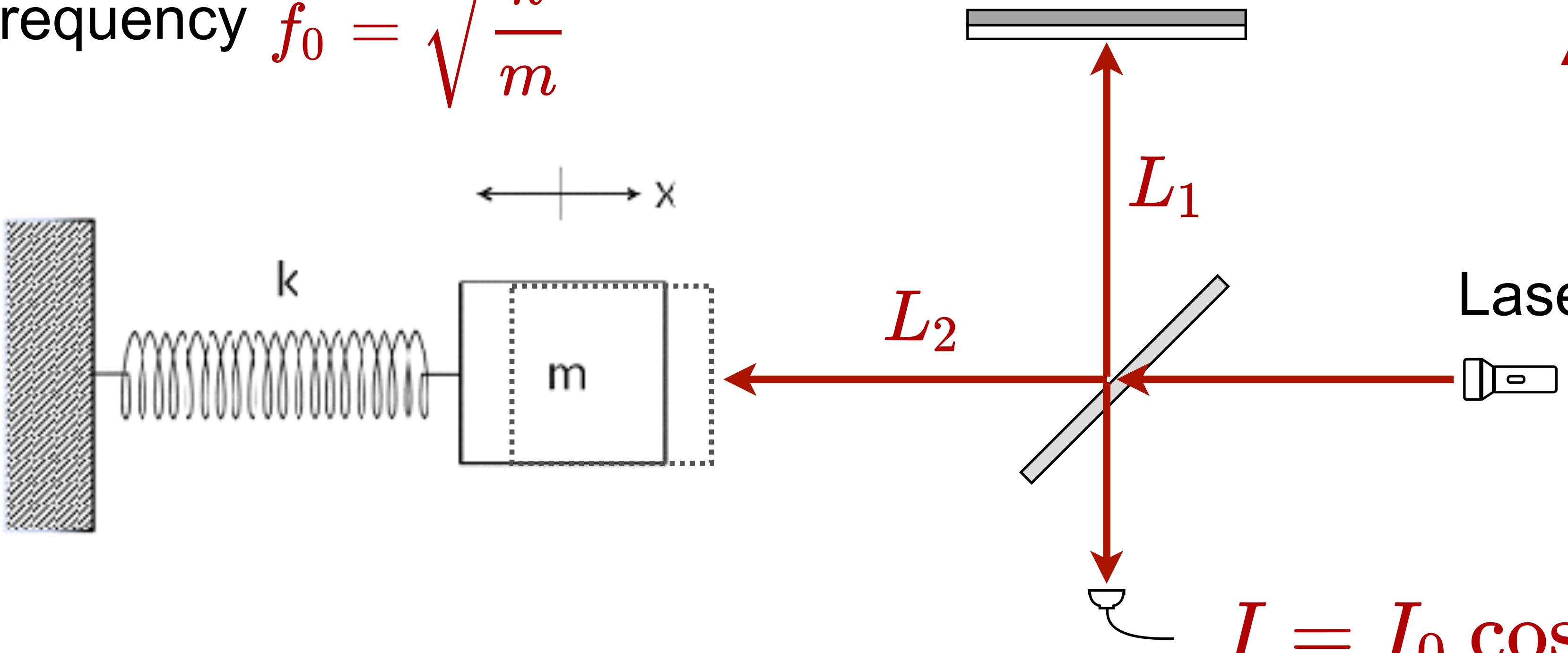


$$\mathrm{std}(\bar{X}_N) = \frac{\sigma}{\sqrt{N}}$$

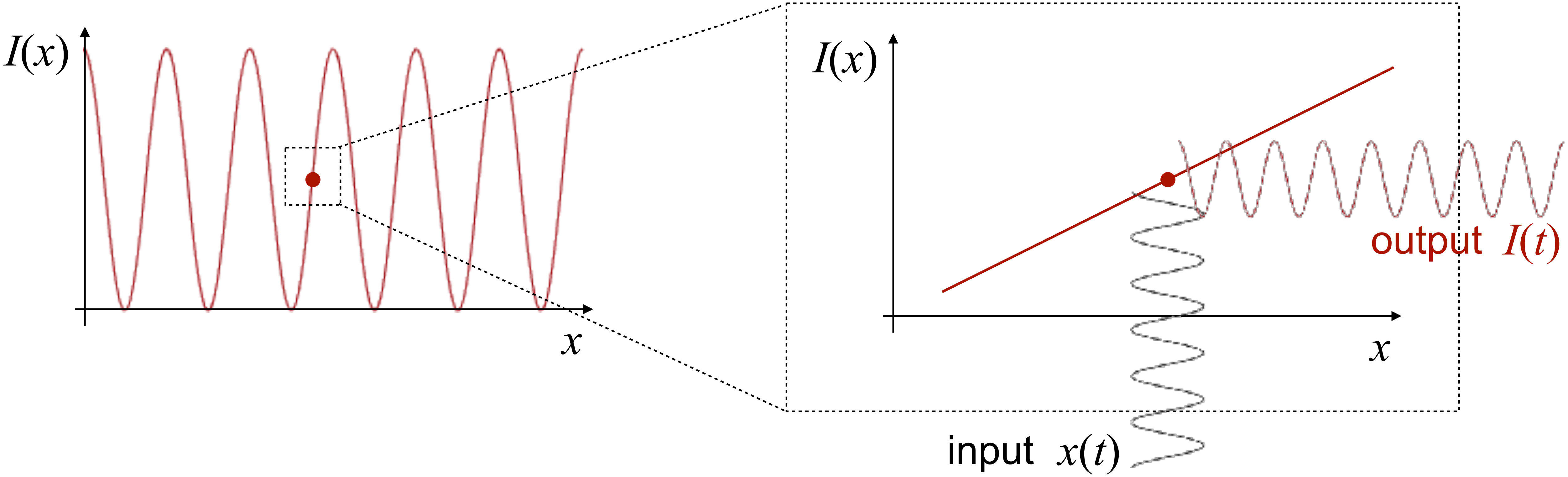
Oscillator Frequency

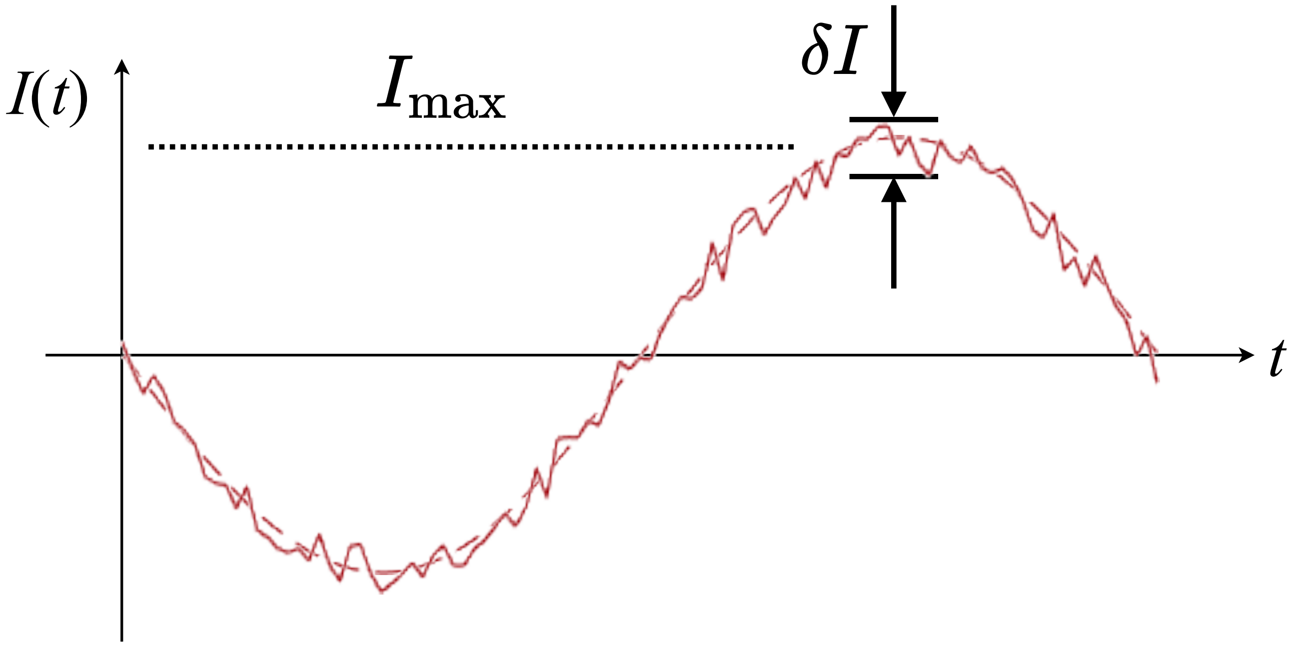
frequency $f_0 = \sqrt{\frac{k}{m}}$

$\Delta L = L_2 - L_1$



$I = I_0 \cos^2(k \cdot \Delta L)$



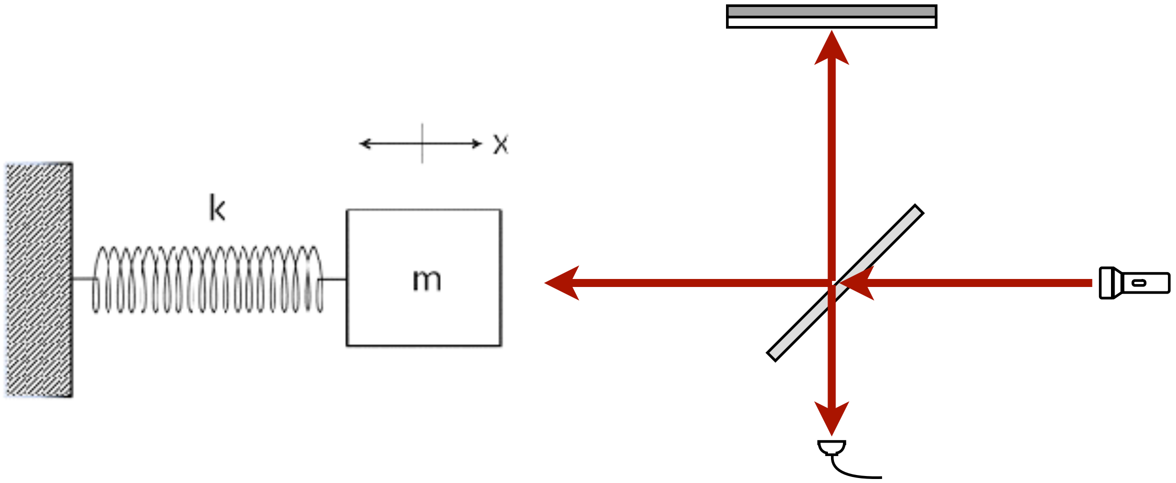


signal: $I(t)$ noise: δI

Signal to Noise Ratio

$$SNR = \frac{I_{\max}}{\delta I}$$

Phase Uncertainty $\delta\phi = SNR^{-1}$



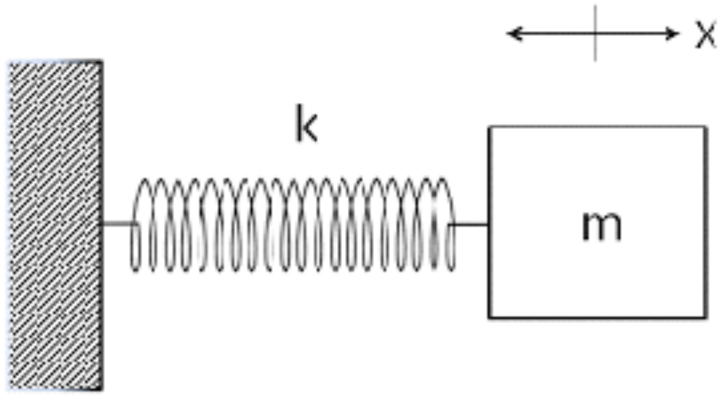
Fundamental noise:

- **laser shot noise**
  - Poisson process
- **oscillator quantum noise**
  - laser ‘disturbs’ the oscillator

- ruler uncertainty
  - signal-to-noise ratio (SNR)

- page vs. book
  - system coherence

Oscillator Frequency

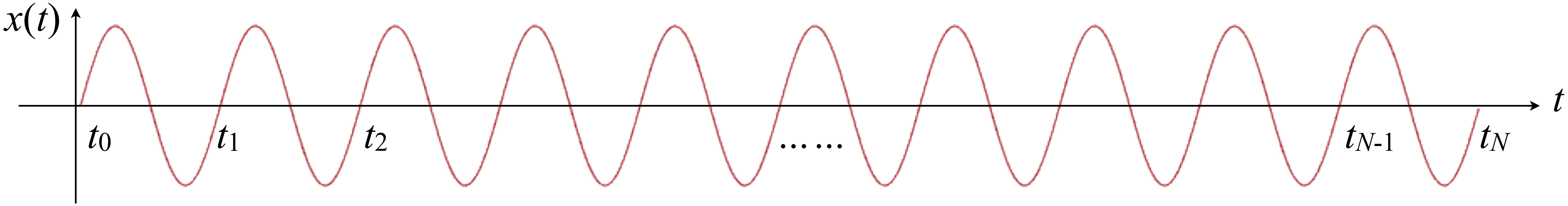


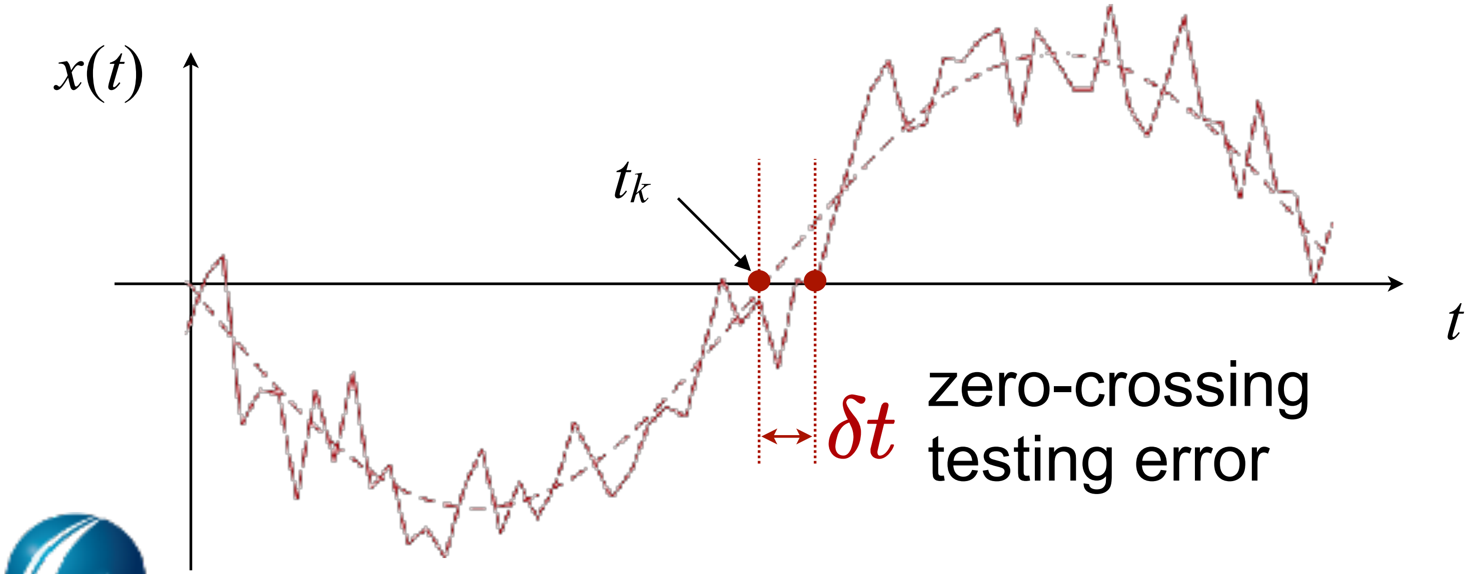
frequency

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$$

period

$$\tau = \frac{1}{f_0}$$





zero-crossing testing error

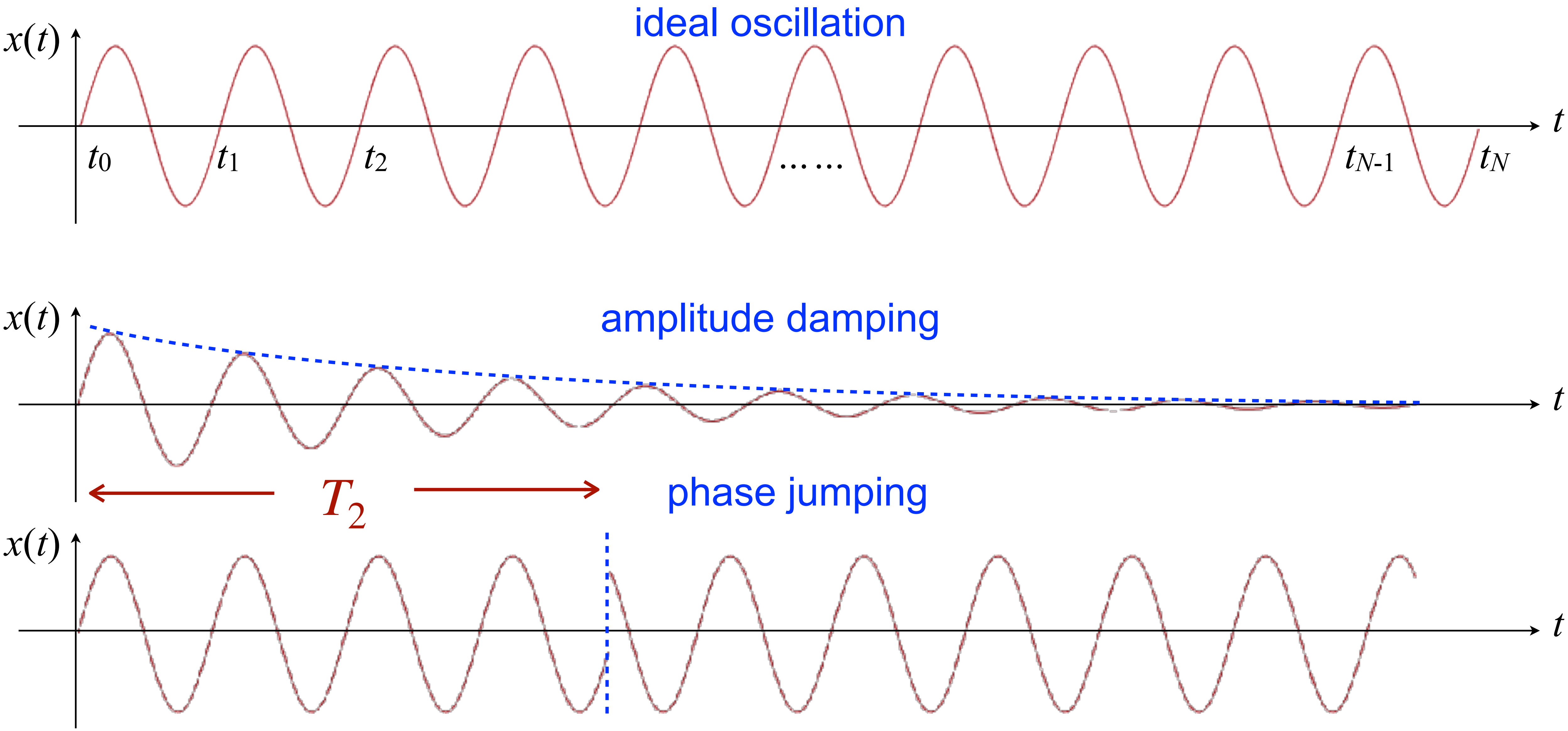
$$\tau = \frac{t_N - t_0}{N} \qquad \delta\tau \sim \frac{\delta t}{N}$$

$$\delta f \sim f_0 \cdot \frac{\delta t}{T}$$

frequency uncertainty

measurement time

Two Decoherence Mechanisms



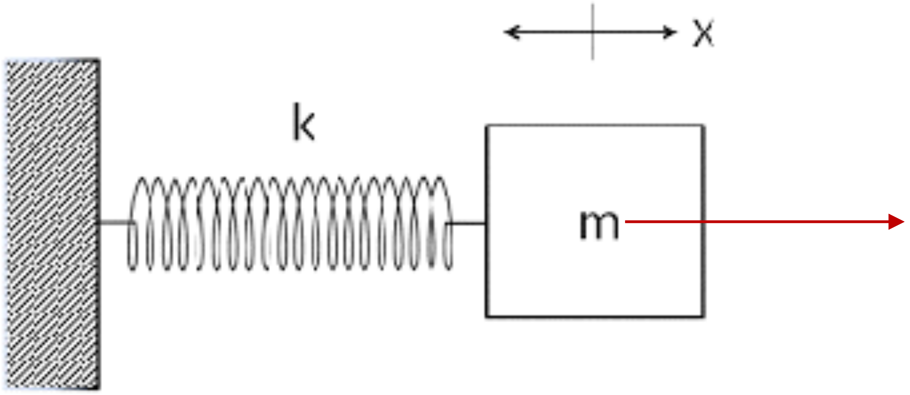
Coherence time T_2 :

characteristic time that the **phase is predictable**.

max. measurement time $T_{\max} \approx T_2$

Coherence time limits the measurement precision.

Forced Oscillator with Damping

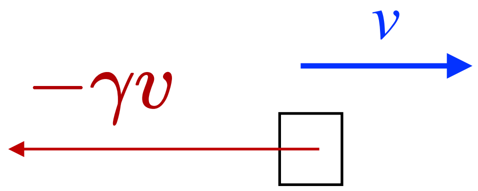


$$F(t) = F_0 \cos \omega t$$

frequency

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$$

$$f = -\gamma v$$



resistance force

$$\ddot{x} + 2\Gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos \omega t$$

damping rate $\Gamma = \gamma/(2m)$

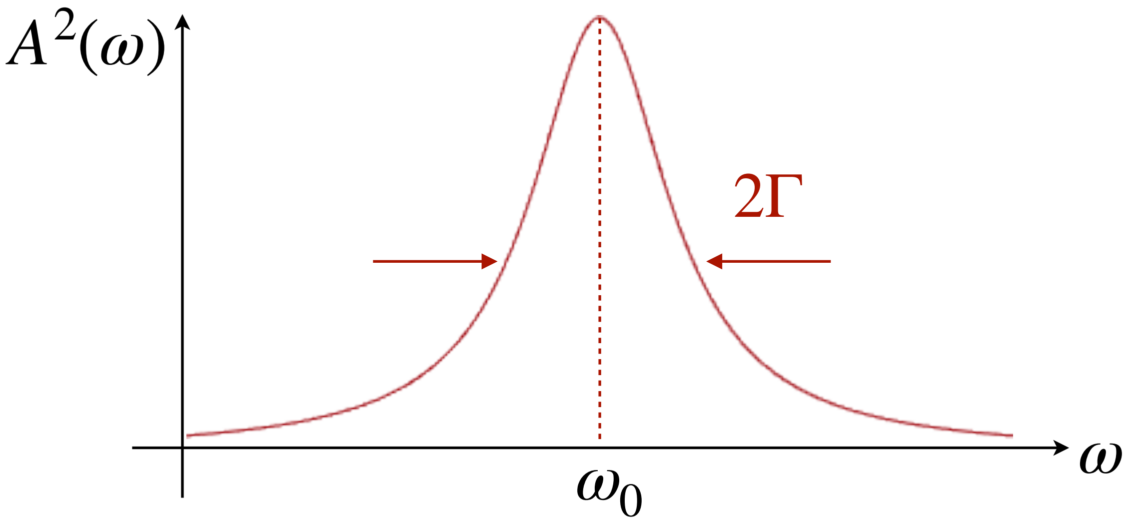
steady state solution

$$x(t) = \frac{\left(\omega_0^2 - \omega^2\right)\cos(\omega t) + 2\Gamma\omega\sin(\omega t)}{4\Gamma^2\omega^2 + \left(\omega^2 - \omega_0^2\right)^2}\frac{F_0}{m}$$

$$x(t) = \boxed{A(\omega)}\cos[\omega t + \boxed{\phi(\omega)}]$$

amplitude

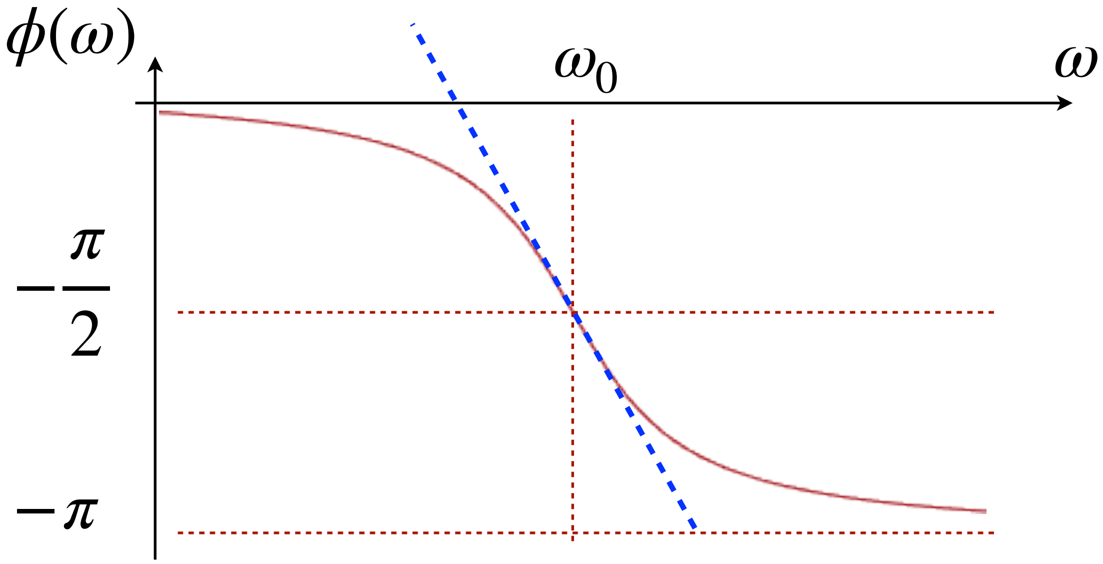
phase



Amplitude-Frequency Response

$$A^2(\omega) \propto \frac{1}{(\omega-\omega_0)^2+\Gamma^2}$$

line width $\Gamma = T_2^{-1}$

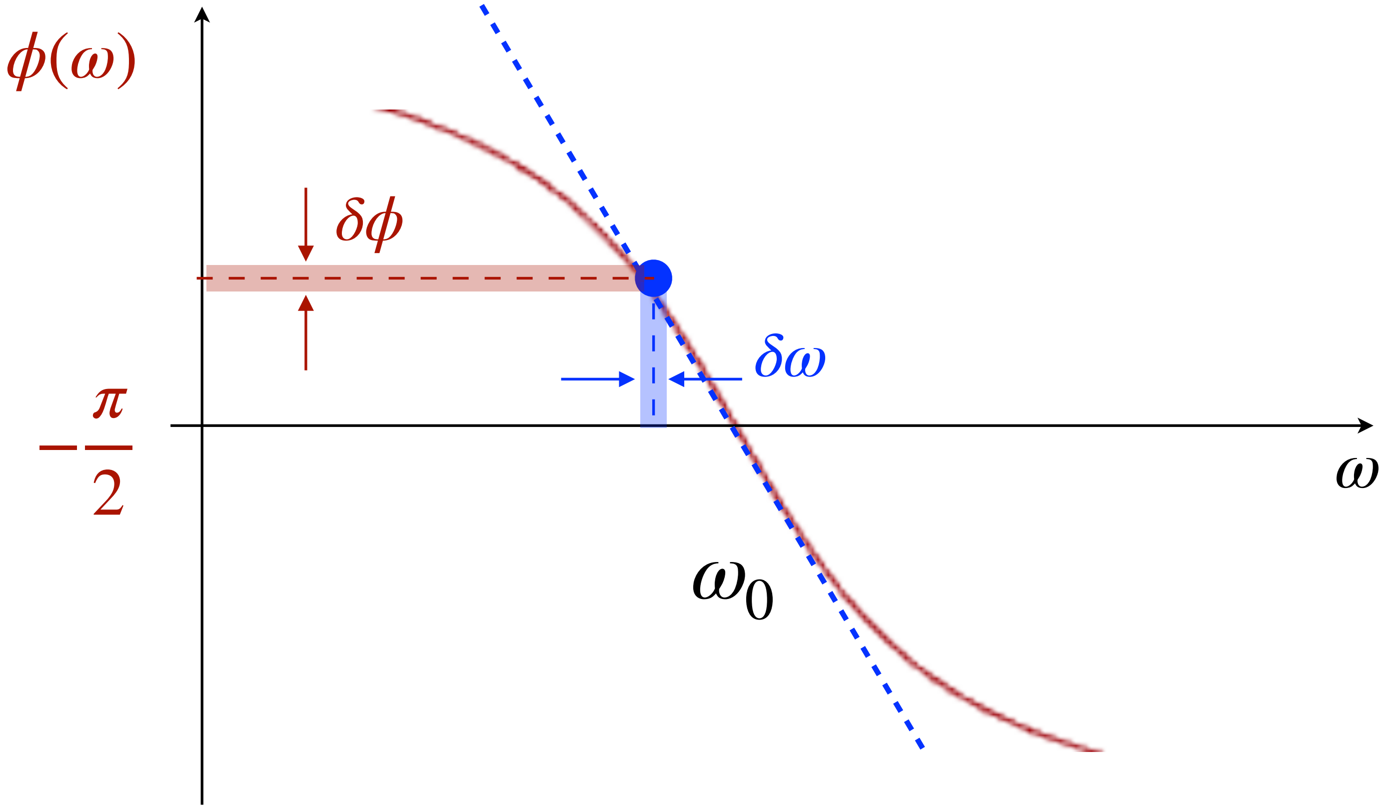


Phase-Frequency Response

$$\phi(\omega) = \arctan\left(\frac{\Gamma}{\omega-\omega_0}\right)$$

Near the resonant point

$$\omega-\omega_0 = -\Gamma\left(\phi+\frac{\pi}{2}\right)$$



Near the resonant point

$$\omega - \omega_0 = -\Gamma\left(\phi + \frac{\pi}{2}\right)$$

frequency $\omega - \omega_0$ is derived

from measured phase ϕ

$$\delta\omega = \Gamma \cdot \delta\phi$$

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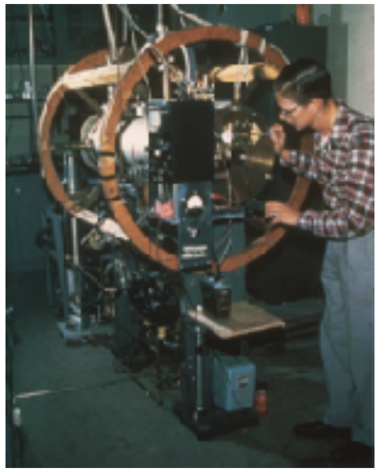
$$\delta\omega = \frac{\Gamma}{SNR}$$

55 cm
300 Hz



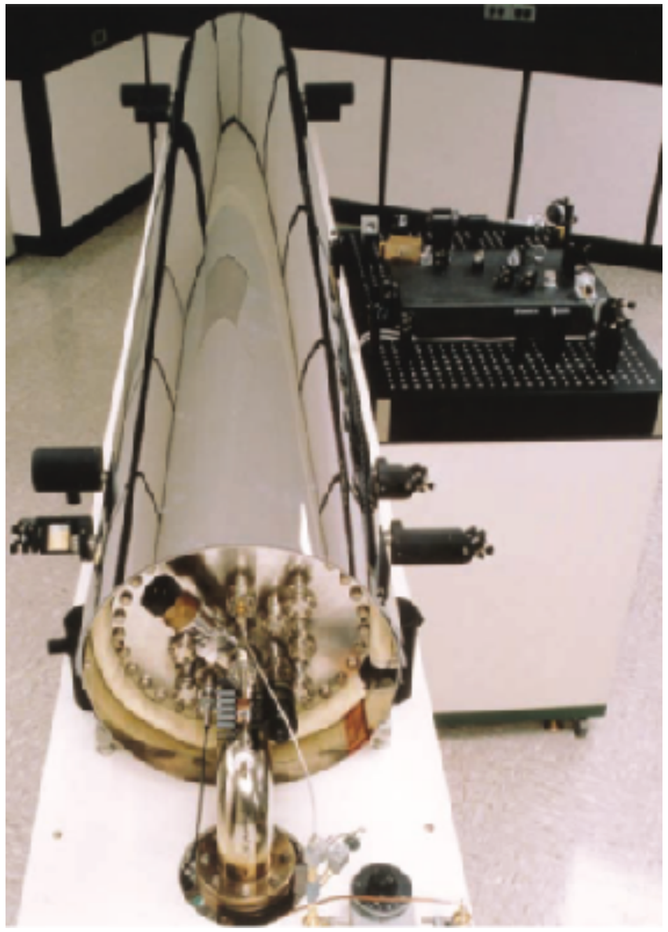
NBS-1, 1959

1×10^{-11}

374 cm
26 Hz

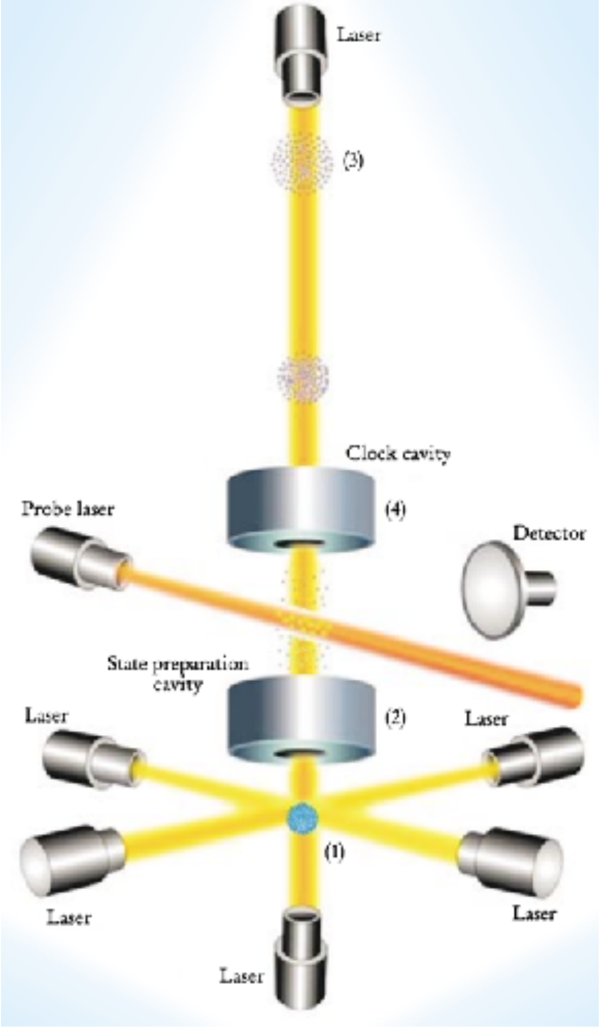
NBS-6, 1975 8×10^{-14}

155 cm, 62 Hz



NIST-7. 1993 5×10^{-15}

~1 m/s



Cs fountain clock

4×10^{-16}

incoherent

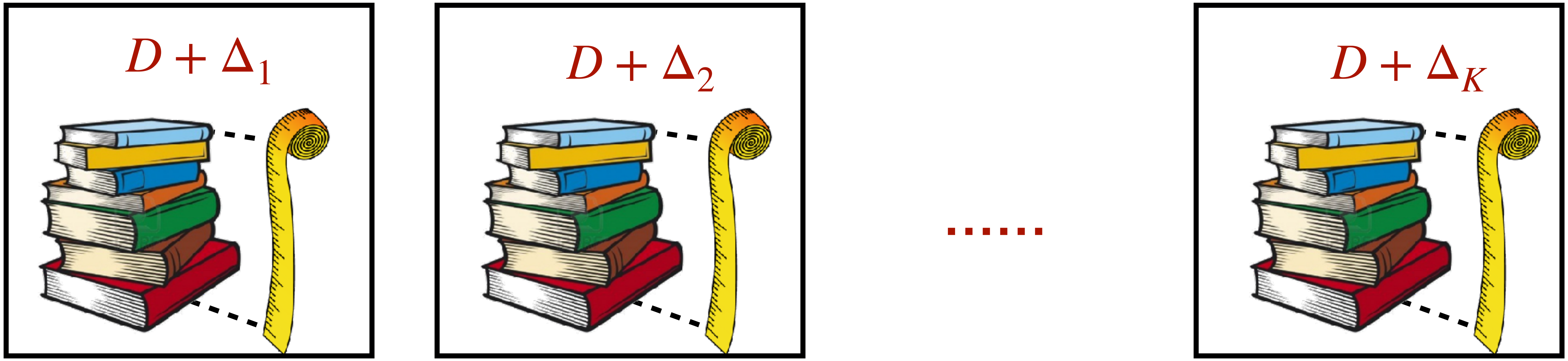
coherent

“Signal” should be accumulated

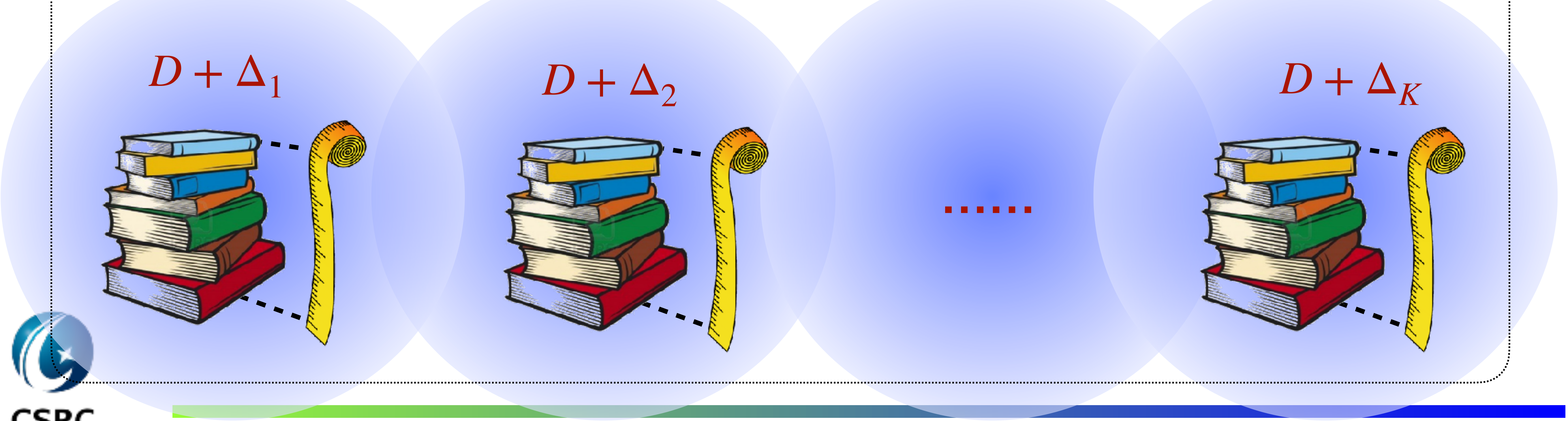
as coherently as possible.

Quantum Entanglement and Quantum Correlation

Classical Ensemble: $\{\Delta_i\}$ are independent random variables



Quantum Ensemble: $\{\Delta_i\}$ are correlated (entangled)



Summary

What is quantum sensing?

- Atomic Clock
- Atom Interferometer
- Optical Magnetometer
- Rydberg Atoms
- Sensing with Quantum Resources

Why do we study quantum sensing?

- High Precision
- Spatial Resolution
- System miniature

How does quantum sensing work?

$$\delta\omega = \frac{\Gamma}{SNR}$$

what is NOT covered in this talk

$$\delta\omega = \frac{\Gamma}{SNR}$$

- what causes the decoherence?
  - atomic collision
  - light scattering
  - inhomogeneous fields
  - ……
- how to increase coherence time?
  - decoupling environment
  - noise compensation
  - …..

- more about quantum resource
  - quantum radar
  - quantum imaging
  - quantum illumination
  - ……
- quantum metrology theory
  - quantum Fisher information
  - Cramér-Rao bound

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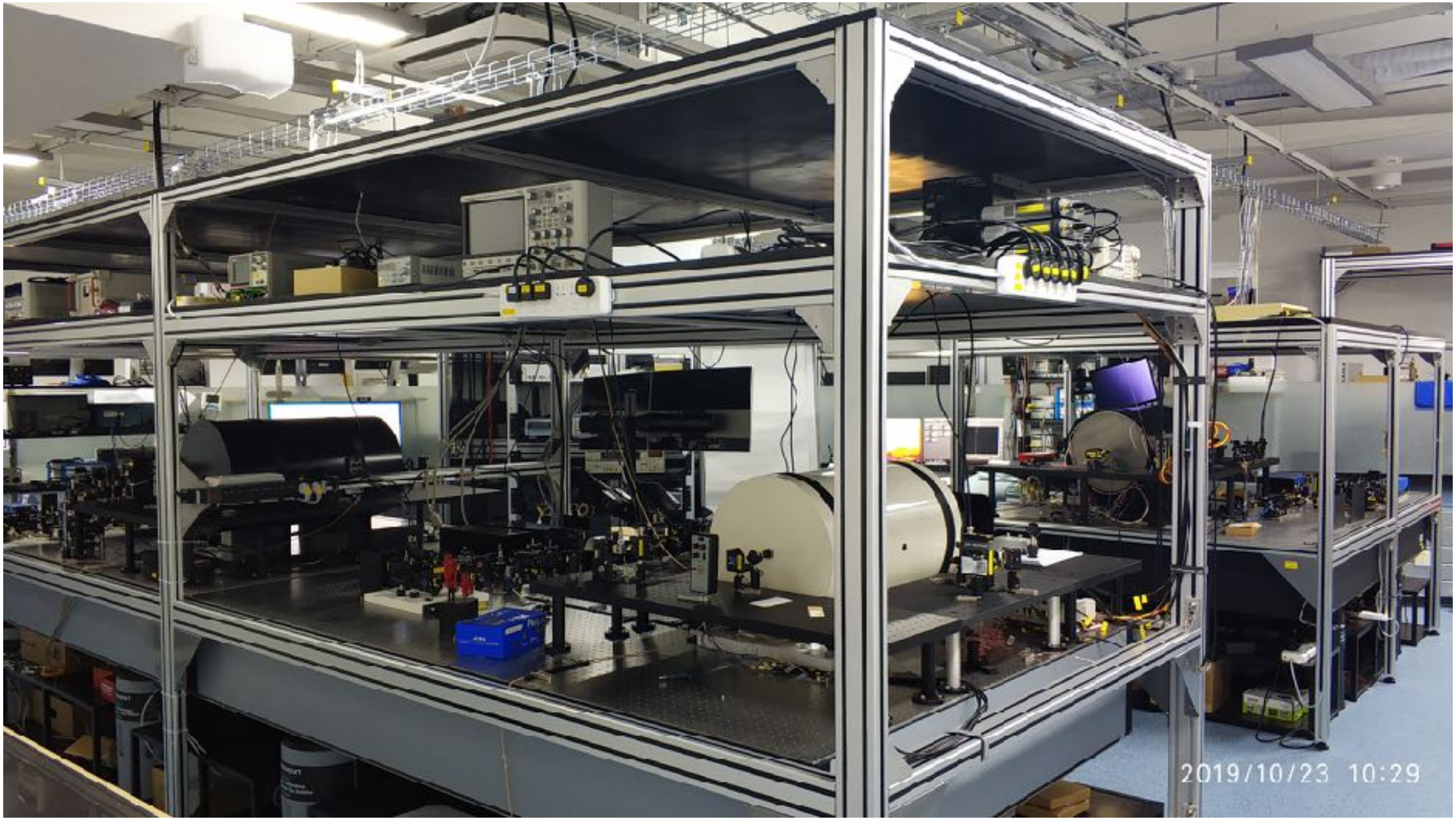
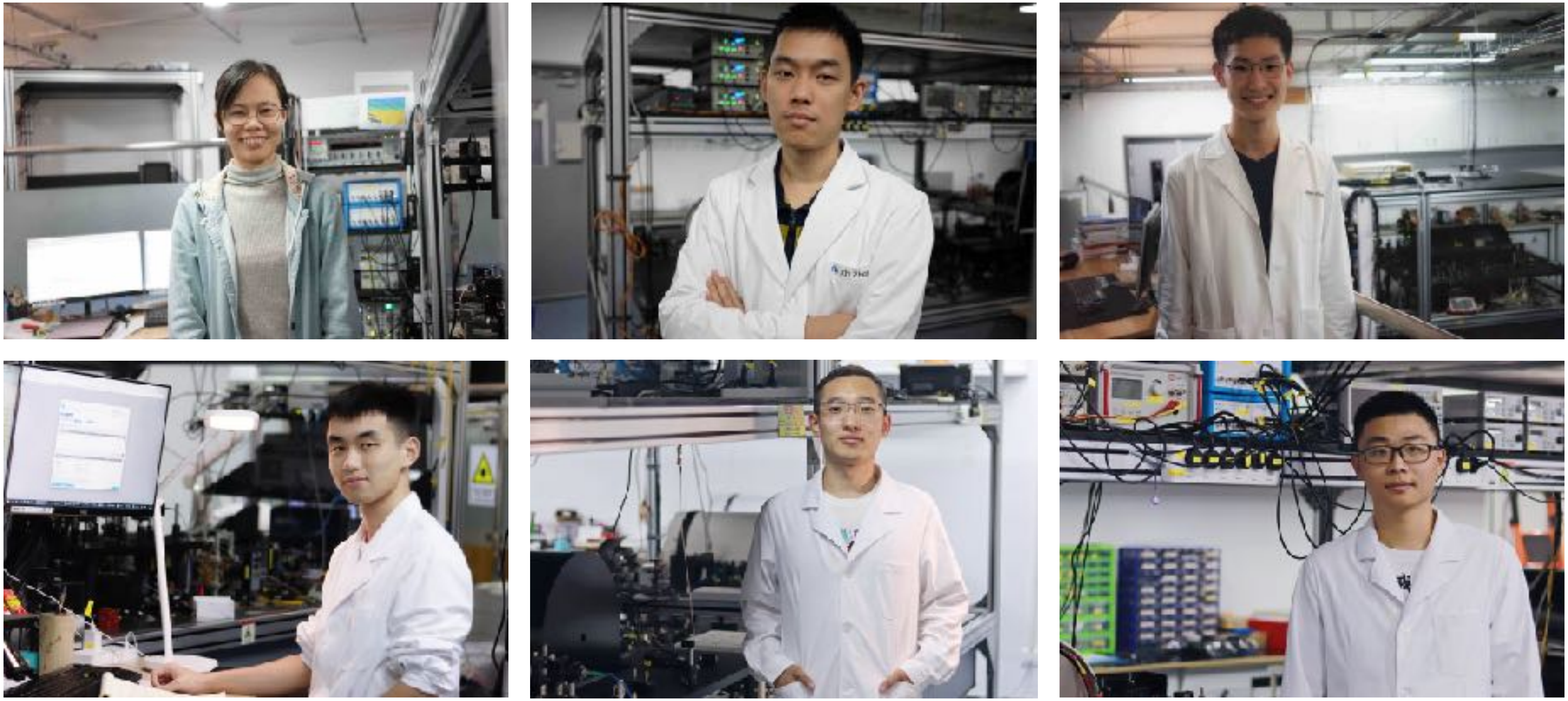


现有研究团队（学生）



欢迎各位同学来访参观

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