Topological Superradiant Phase in a Degenerate Fermi Gas

Wei Zhang（张威）
Department of Physics
Renmin University of China, Beijing

CSRC Workshop on Quantum Non-Equilibrium Phenomena: Methods and Applications
06/21/2019

Acknowledgements
Jian-Song Pan (USTC)
Wei Yi (USTC)
Xiong-Jun Liu (PKU)
Dong-Yang Yu (IOP)

Outline

- Introduction
  - Superradiance
  - Spin-orbit coupling
  - Cavity-induced SOC
- Superradiance of a quasi-1D degenerate Fermi gas
  - Superradiance transition
  - Topological properties
  - Phase diagram
  - Experimental signatures
- Summary

Superradiance with two-level emitters

PHYSICAL REVIEW — VOLUME 93, NUMBER 1 — JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

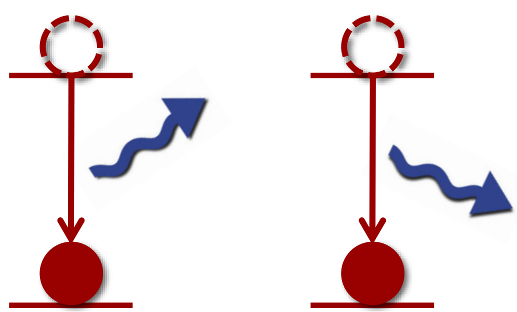
R. H. Dicke

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received August 25, 1953)

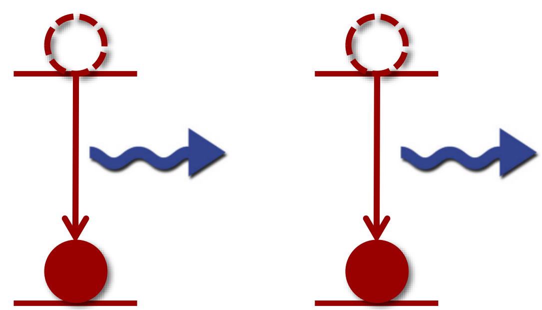
Spontaneous Emission

$I \sim N$

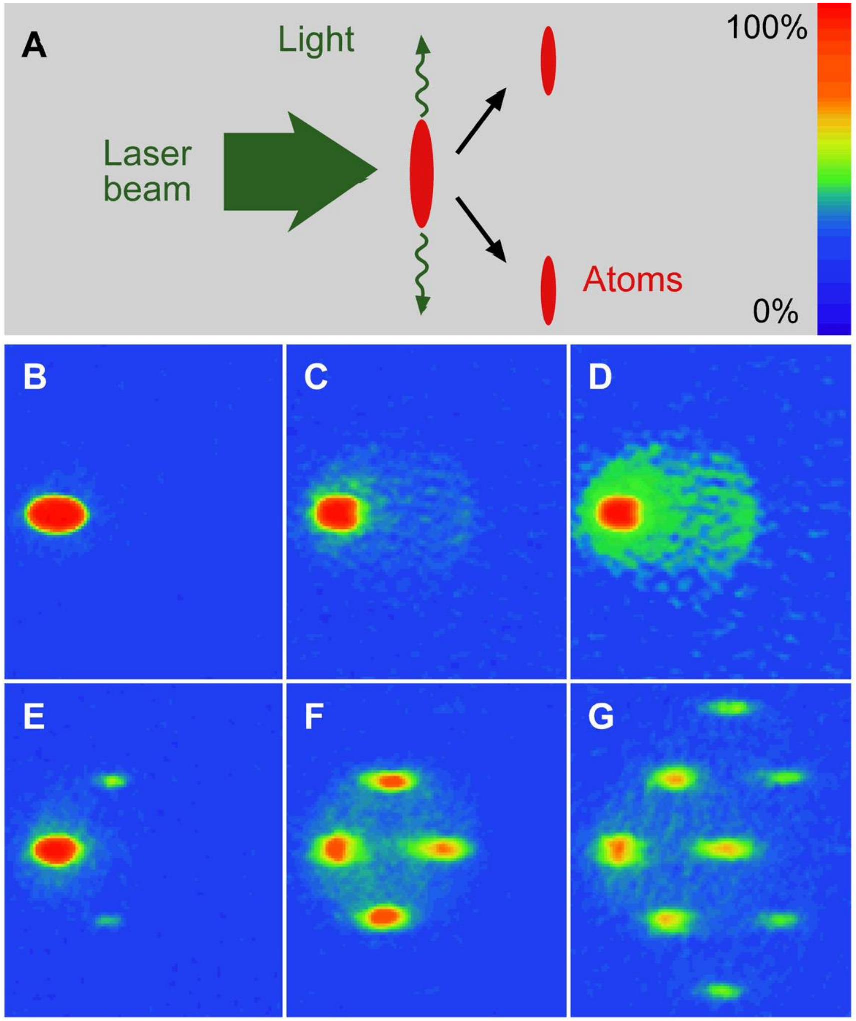


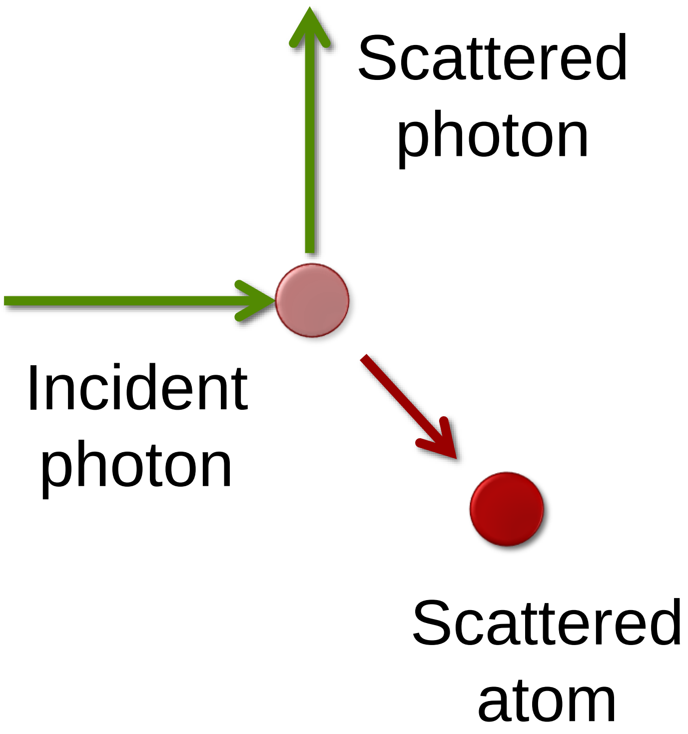
Spontaneous Emission

$I \sim N^2$



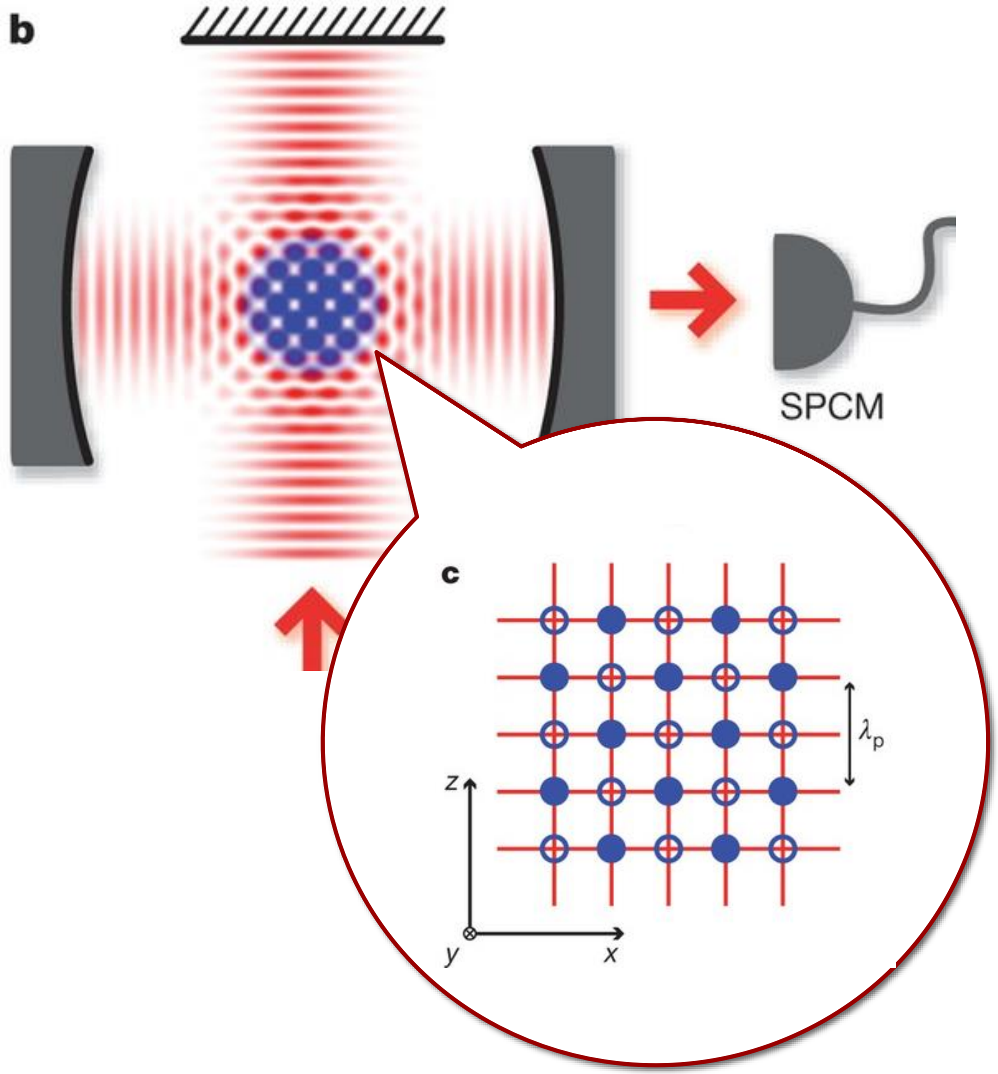
Superradiance with BEC

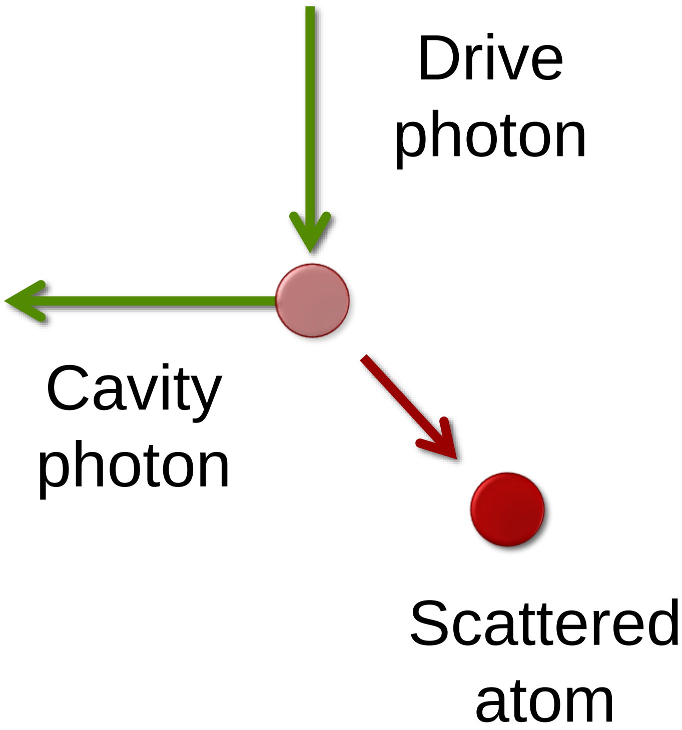




Inouye *et al.*, Science 285, 571 (1999)

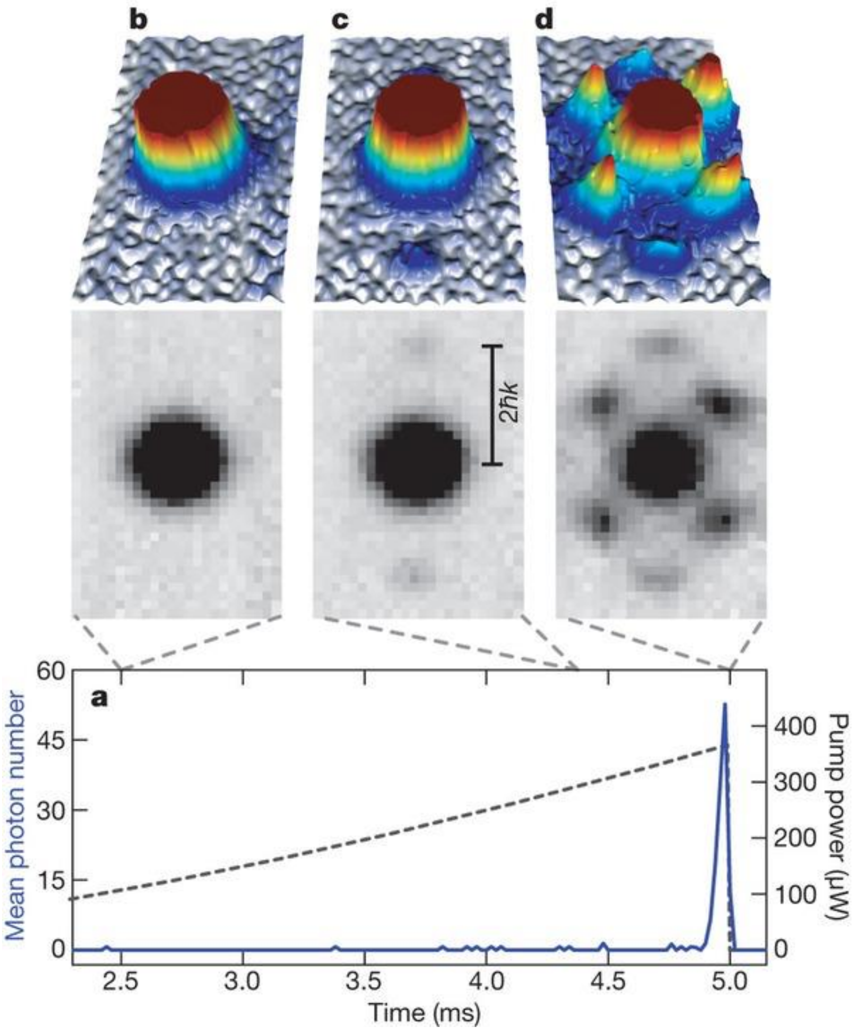
Superradiance with BEC in a cavity

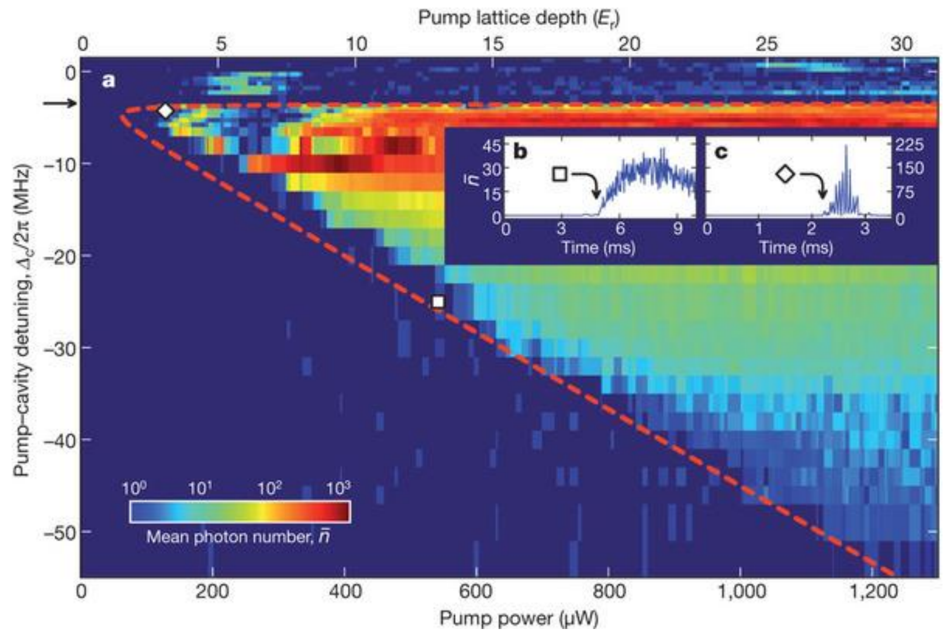




Baumann *et al.*, Nature 464, 1301 (2010)

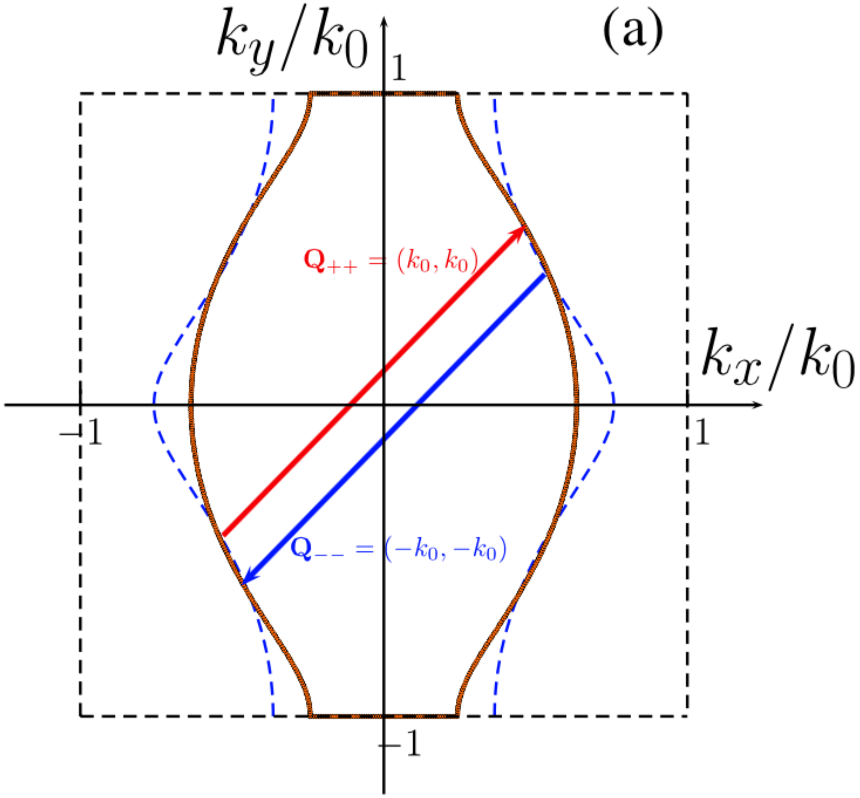
Superradiance with BEC in a cavity

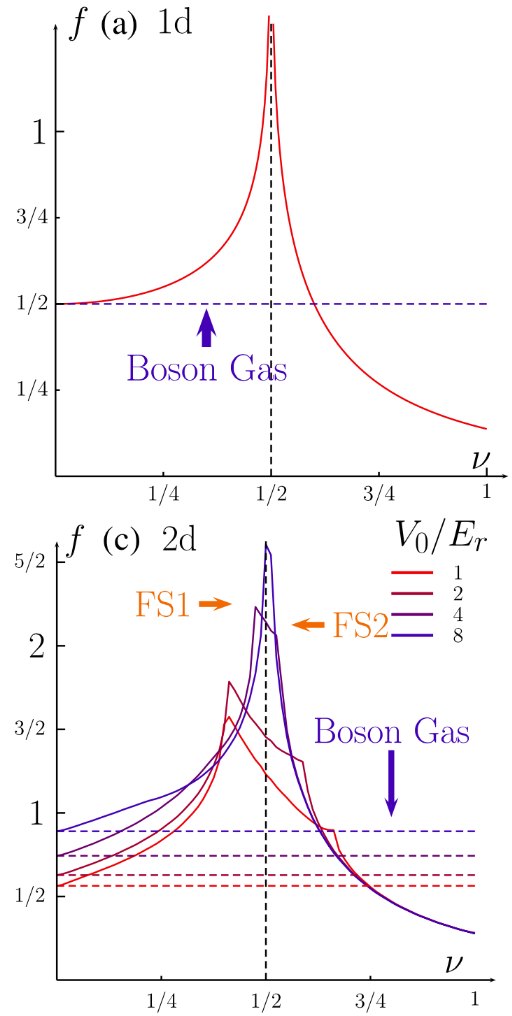




Baumann *et al.*, Nature 464, 1301 (2010)

Superradiance with a degenerate Fermi gas

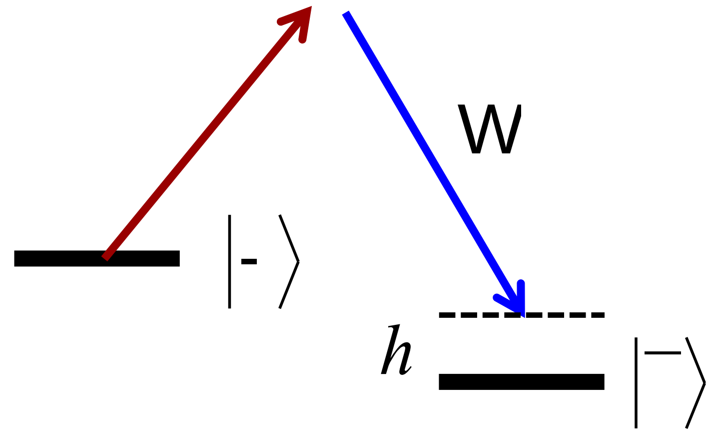




Keeling *et al.*, PRL 112 143002, (2014)
Piazza *et al.*, PRL 112 143003, (2014)
Chen *et al.*, PRL 112 143004, (2014)

Synthetic spin-orbit coupling: Raman scheme

Lin et. al., Nature 471, 83 (2011)



$$|\text{-}\rangle \qquad |\bar{\ }\rangle$$

$$H = \begin{pmatrix} \frac{k_x^2}{2m} + \frac{h}{2} & \frac{\Omega}{2} e^{i2k_0x} \\ \frac{\Omega}{2} e^{-i2k_0x} & \frac{k_x^2}{2m} - \frac{h}{2} \end{pmatrix} \begin{matrix} |\text{-}\rangle \\ |\bar{\ }\rangle \end{matrix} \qquad U = \begin{pmatrix} e^{-ik_0x} & 0 \\ 0 & e^{ik_0x} \end{pmatrix}$$

Synthetic spin-orbit coupling: Raman scheme

$$H_{\mathrm{SO}} = UHU^{\dagger} = \begin{pmatrix} \frac{(k_x+k_0)^2}{2m} + \frac{h}{2} & \frac{\Omega}{2} \\ \frac{\Omega}{2} & \frac{(k_x-k_0)^2}{2m} - \frac{h}{2} \end{pmatrix}$$

$$= \frac{1}{2m}(k_x + k_0\sigma_z)^2 + \frac{\Omega}{2}\sigma_x + \frac{h}{2}\sigma_z.$$

- Spin rotation, x to –z, z to x

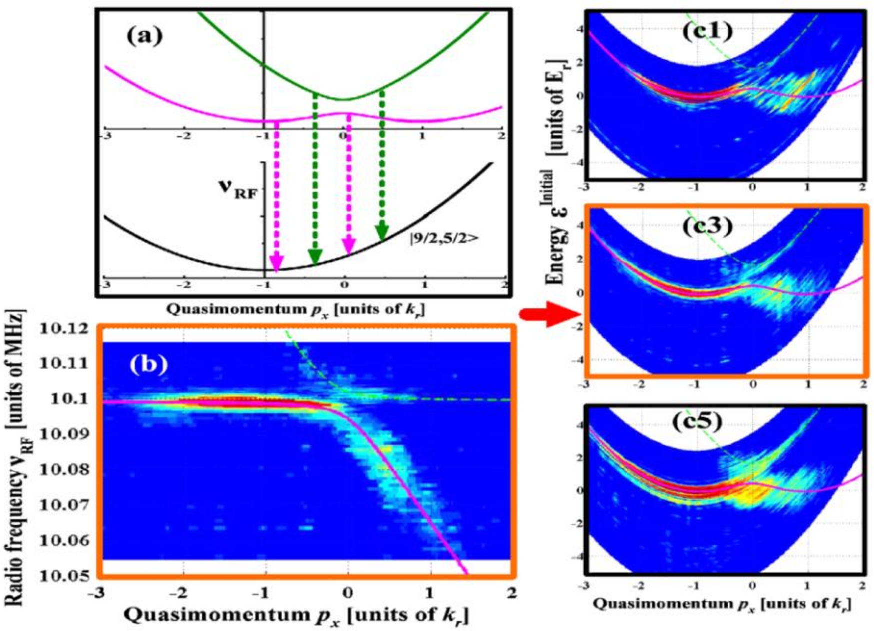
$$H_{\mathrm{SO}} = \frac{1}{2m}\left(k_x^2 + k_0^2 + 2s_x k_x\right) - \frac{W}{2}s_z + \frac{h}{2}s_x$$

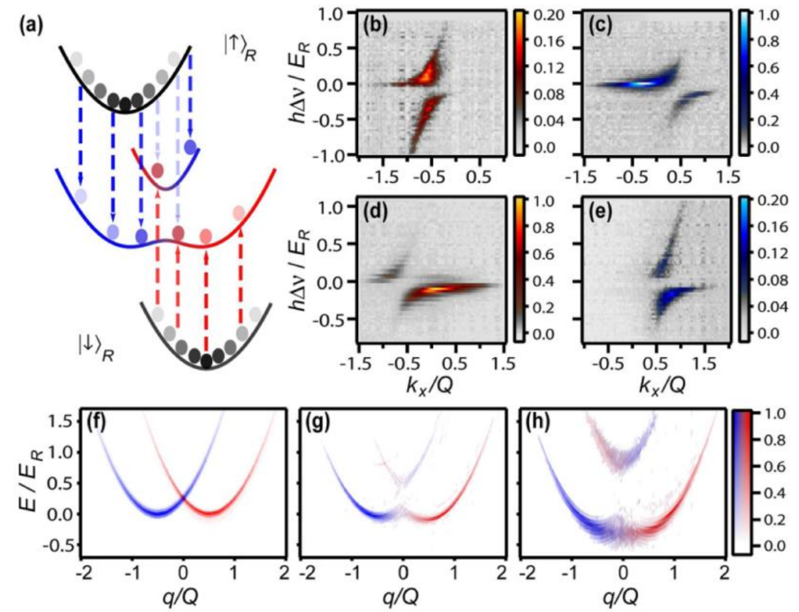
1D (ERD) SOC

x-field
2-photon
detuning

z-field
Rabi Frequency

Implementation in Fermi gases





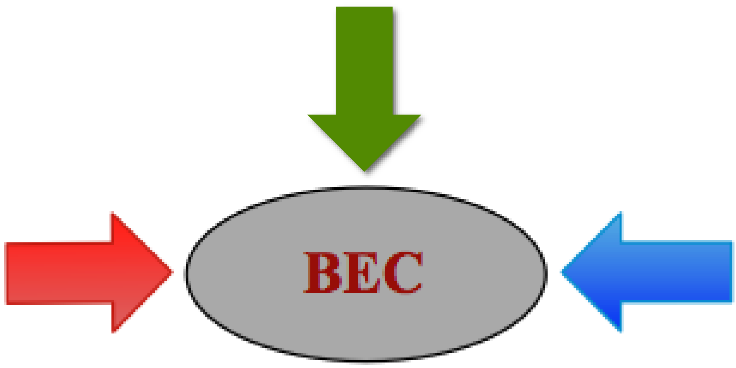
Shanxi Univerisity, Taiyuan, China

Wang *et al.*, PRL 109, 095301 (2012)

MIT, Boston, USA

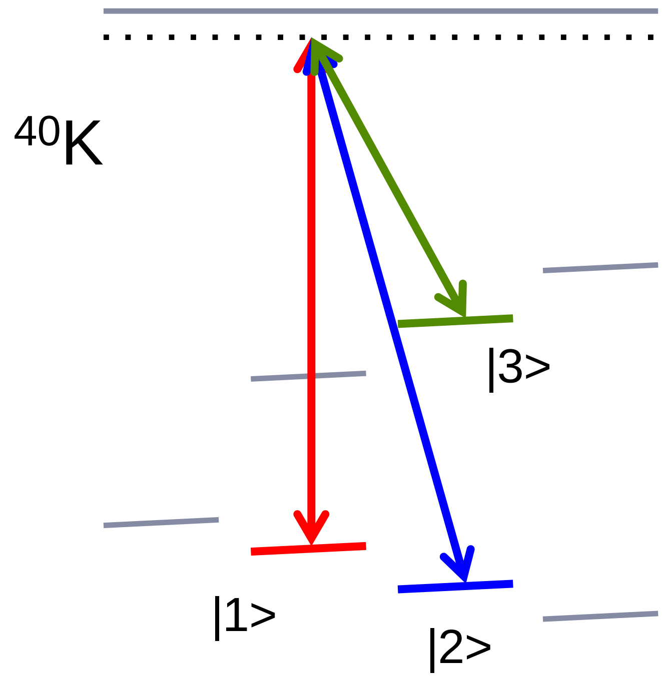
Cheuk *et al.*, PRL 109, 095302 (2012)

Implementation of 2D SOC



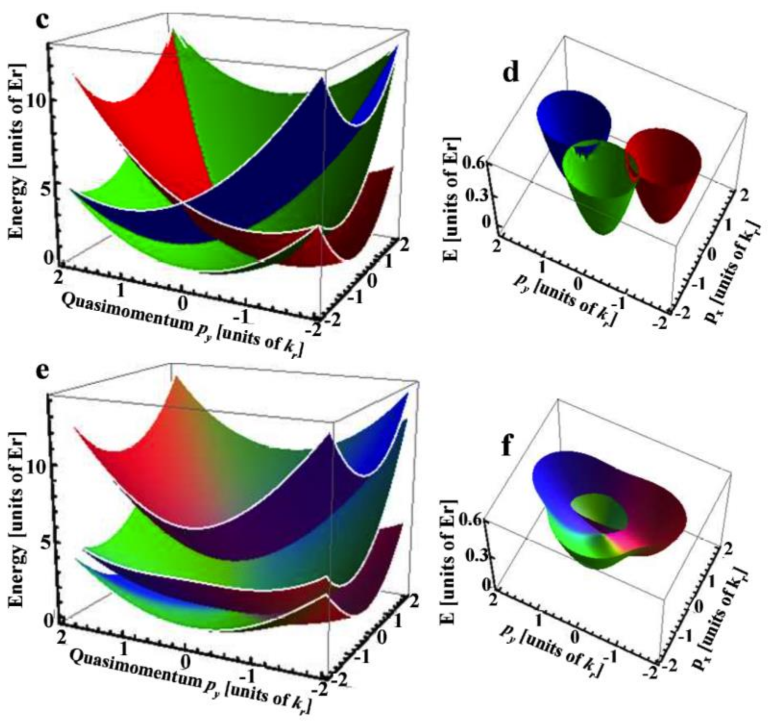
Shanxi Univerisity, Taiyuan, China

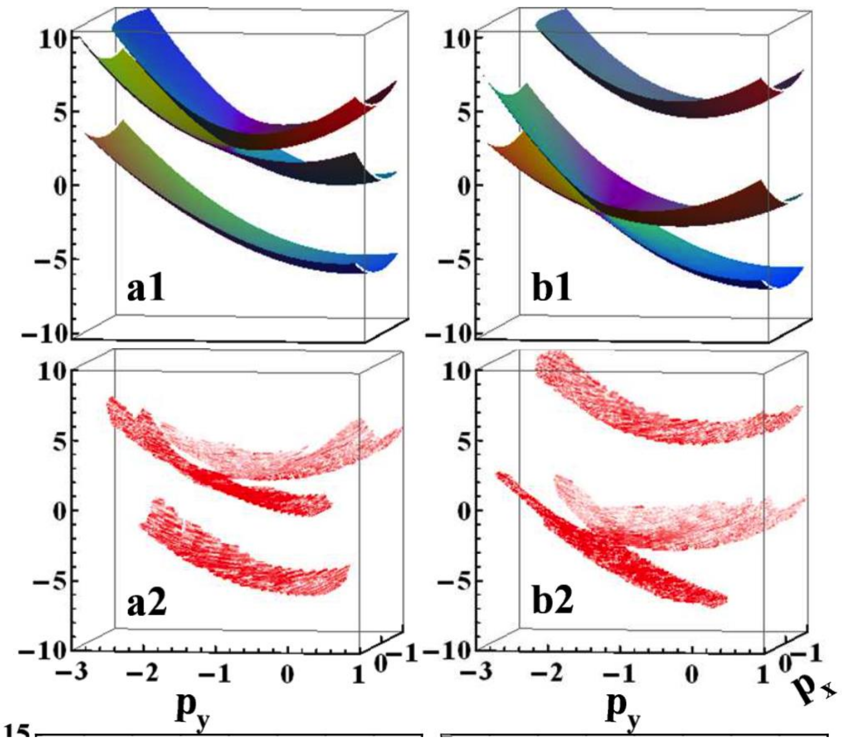
Huang *et al.*, arXiv:1506.02861 (2015)



$$H = \begin{pmatrix} (\mathbf{p} - \mathbf{k}_1)^2 + d_1 & -W_{12}/2 & -W_{13}/2 \\ -W_{12}/2 & (\mathbf{p} - \mathbf{k}_2)^2 + d_2 & -W_{23}/2 \\ -W_{13}/2 & -W_{23}/2 & (\mathbf{p} - \mathbf{k}_3)^2 + d_3 \end{pmatrix} + k_z^2$$

Implementation of 2D SOC





Shanxi Univerisity, Taiyuan, China

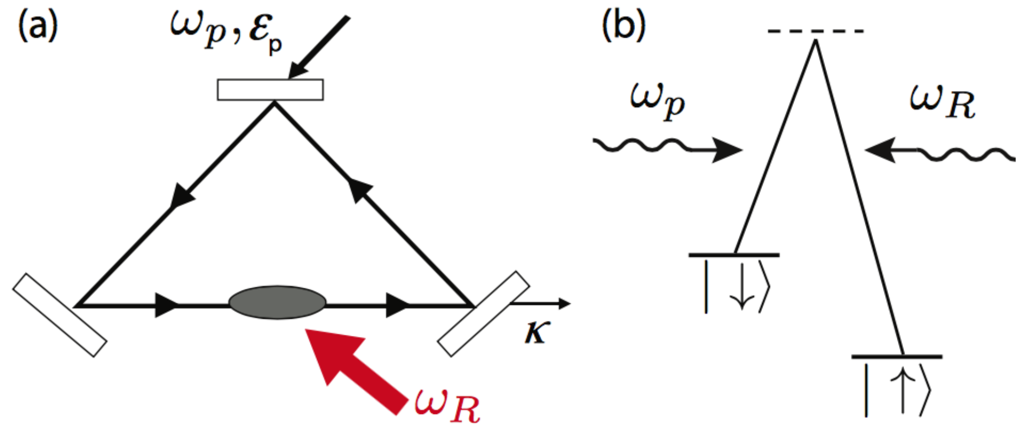
Huang *et al.*, arXiv:1506.02861 (2015)

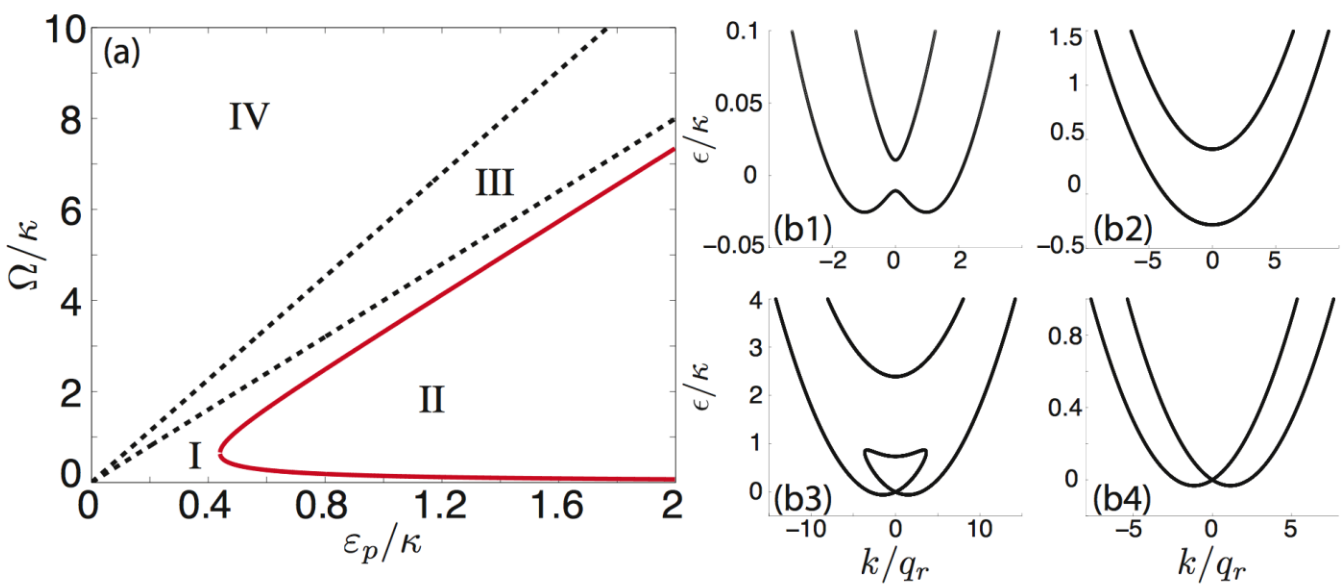
$$H_{SO} = a_x s_x k_x + a_y s_y k_y + a_z s_z k_z$$

$a_x = a_y = a_z$	**3D isotropic SOC**	$a\vec{s} \times \vec{k}$
$a_x = a_y$ $a_z = 0$	**Rashba SOC**	$a\left(s_x k_x + s_y k_y\right)$
$a_x = - a_y$ $a_z = 0$	**Dresselhaus SOC**	$a\left(s_x k_x - s_y k_y\right)$
$a_y = a_z = 0$	**1D SOC** **Equal R-D SOC**	$a s_x k_x$

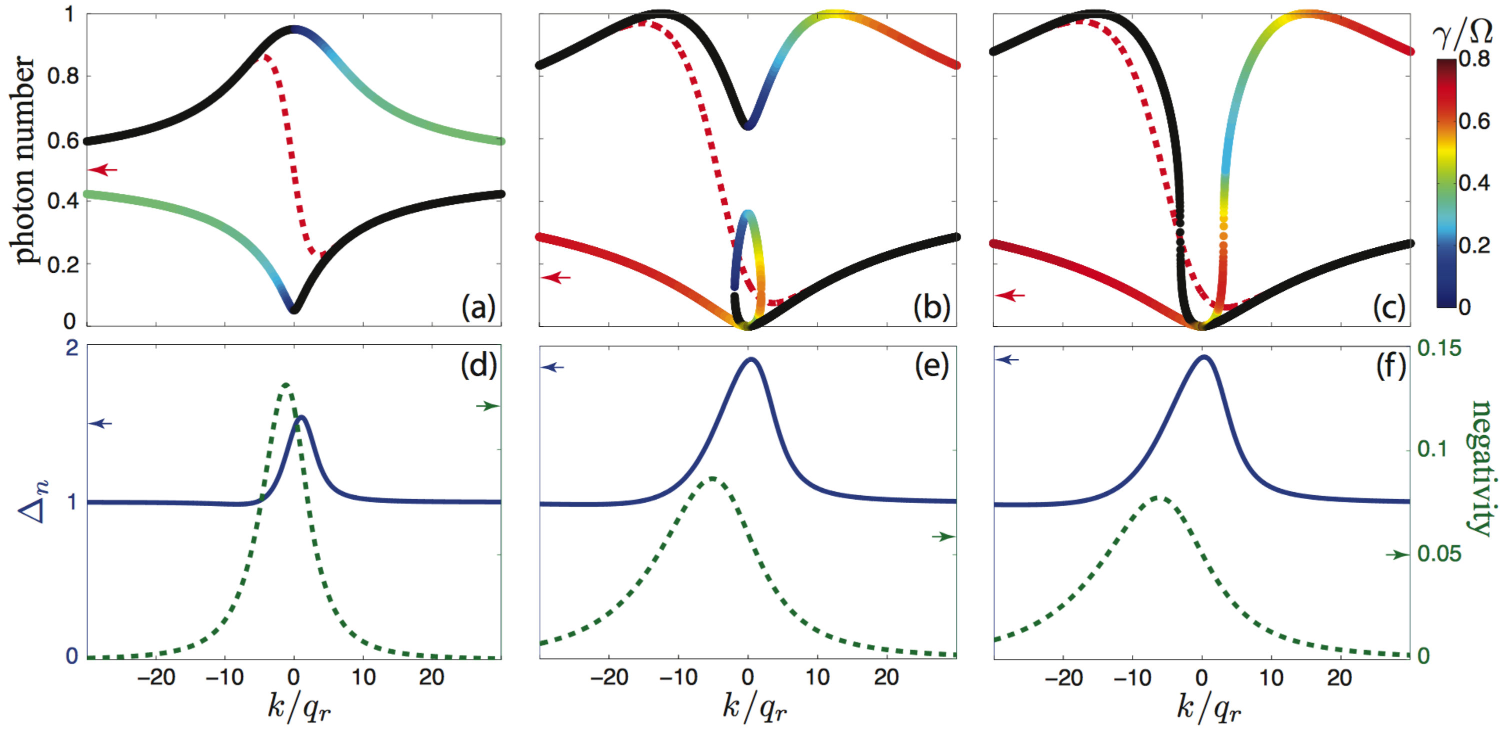
Cavity-induced SOC

Dong, Zhou, Wu, Ramachandhran, Pu
PRA 89, 011602(R) (2014);
Dong, Zhu, Pu, Atom 3, 183 (2015)



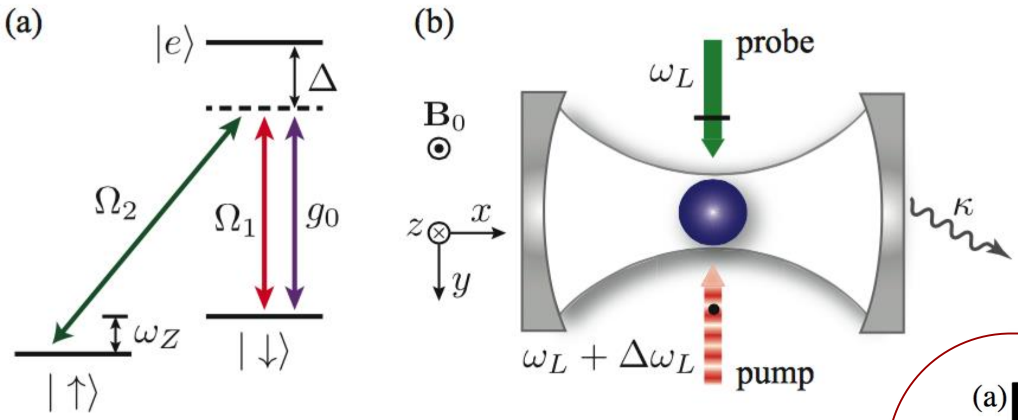


Cavity-induced SOC

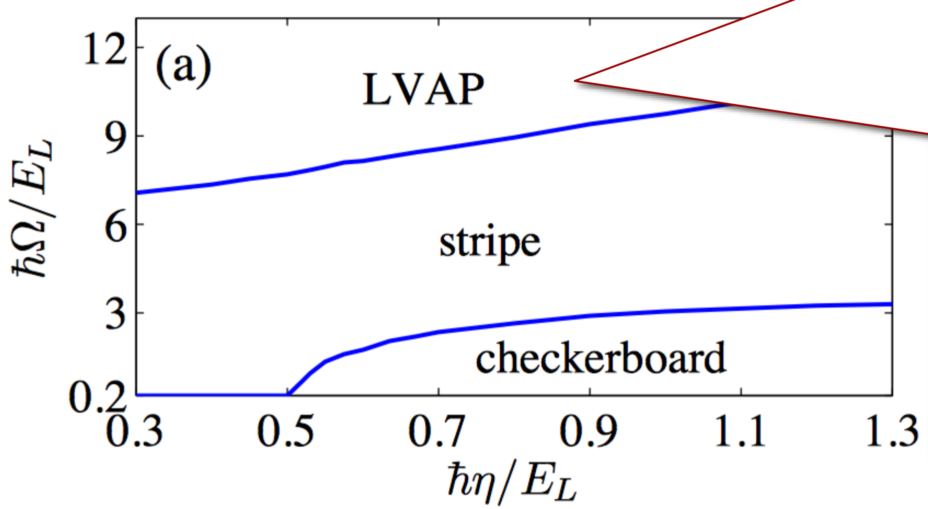


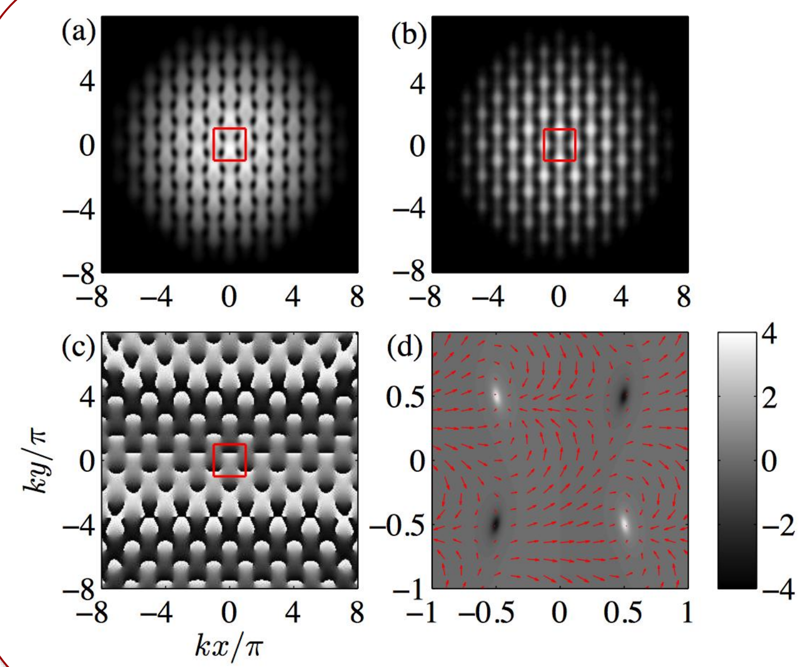
Dong, Zhu, Pu, Atom 3, 183 (2015)

Cavity-induced SOC



Deng, Cheng, Jing, Yi,
PRL 112, 143007 (2014)



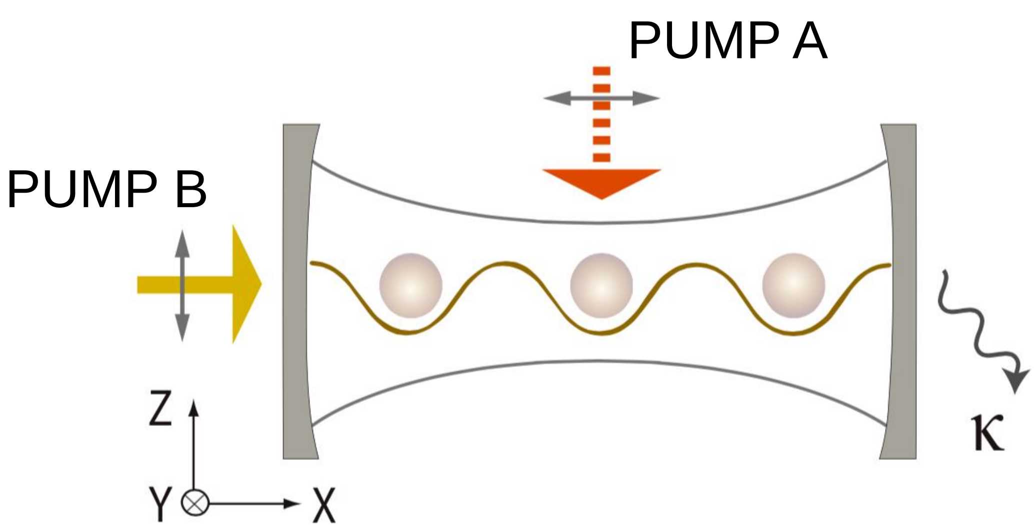


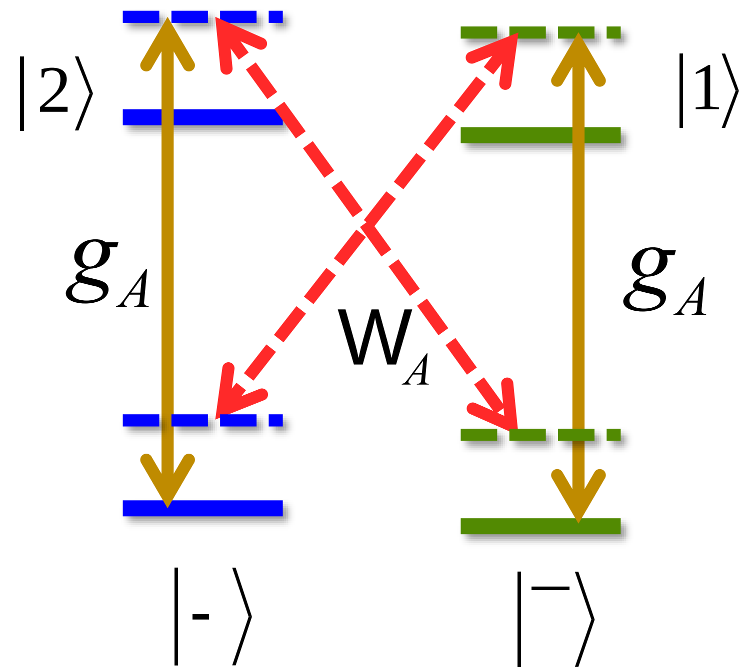
Outline

- Introduction
  - Superradiance of degenerate quantum gases
  - Spin-orbit coupling
  - Cavity-induced SOC
- Superradiance of a 1D degenerate Fermi gas
  - Superradiance transition
  - Topological properties
  - Phase diagram
  - Experimental signatures
- Summary

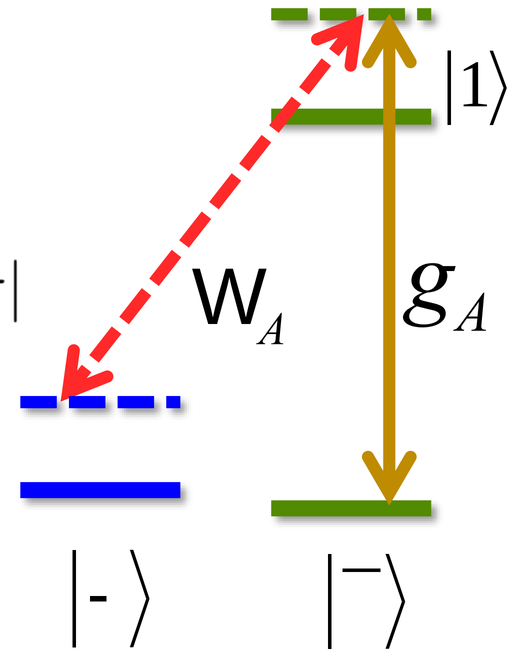
System

- A quasi-1D two-component Fermi gas in a cavity





Effective Hamiltonian



- Dynamics of a single particle $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$

$$\mathcal{H}_0 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \sum_{j=1,2} \hbar\omega_j |j\rangle\langle j| + \sum_{\sigma=\uparrow,\downarrow} \hbar\omega_\sigma |\sigma\rangle\langle\sigma|$$

$$\mathcal{H}_I = -\left(\Omega_{1\uparrow}\sigma^+_{1\uparrow} + \Omega_{2\downarrow}\sigma^+_{2\downarrow}\right) e^{-i(\omega_A t + k_0 z)}$$
$$-\left(g_{1\downarrow}\sigma^+_{1\downarrow}\hat{a} + g_{2\uparrow}\sigma^+_{2\uparrow}\hat{a}\right)\cos\left(k_0 x\right) + h.c.$$

- Unitary transformation $U\left(t\right) = \exp\left[i\left(\sum_{j=1,2}|j\rangle\langle j| + \hat{a}^\dagger\hat{a}\right)\omega_A t\right]$

$$\mathcal{H}_{eff} = -\left[\Delta_A - \xi_A \cos^2\left(k_0 x\right)\right]\hat{a}^\dagger\hat{a} + \hbar m_z \sigma_z$$
$$+\eta\left[\left(\hat{a}e^{ik_0 z} + \hat{a}^\dagger e^{-ik_0 z}\right)|\uparrow\rangle\langle\downarrow| + \text{H.C.}\right]\cos\left(k_0 x\right)$$

Effective Hamiltonian

- Transversal degrees of freedom

$$\hat{H} = \sum_{\sigma} \int d\boldsymbol{r} \hat{\Psi}_{\sigma}^{\dagger} \left[\frac{\boldsymbol{p}^2}{2m} + U(\boldsymbol{r}) + \left(V_0 + \xi_A \hat{a}^{\dagger} \hat{a} \right) \cos^2 (k_0 x) + \xi_{\sigma} m_z \right] \hat{\Psi}_{\sigma} - \Delta_A \hat{a}^{\dagger} \hat{a}$$
$$+ \eta \left[\int d\boldsymbol{r} \hat{\Psi}_{\uparrow}^{\dagger} \left(\hat{a} e^{i k_0 z} + \hat{a}^{\dagger} e^{-i k_0 z} \right) \cos (k_0 x) \hat{\Psi}_{\downarrow} + h.c. \right].$$

- 1D model

$$\hat{H} = \sum_{\sigma} \int dx \hat{\psi}_{\sigma}^{\dagger} \left[\frac{p_x^2}{2m} + \left(V_0 + \xi_A \hat{a}^{\dagger} \hat{a} \right) \cos^2 (k_0 x) + \xi_{\sigma} m_z \right] \hat{\psi}_{\sigma}$$
$$+ \eta_A \left(\hat{a} + \hat{a}^{\dagger} \right) \left[\int dx \hat{\psi}_{\uparrow}^{\dagger} \cos (k_0 x) \hat{\psi}_{\downarrow} + h.c. \right],$$

Effective Hamiltonian

- Equation of motion

$$i\dot{\hat{a}} = \hat{a}\left[\xi_A \sum_\sigma \int dx \hat{\psi}_\sigma^\dagger \cos^2\left(k_0 x\right) \hat{\psi}_\sigma - \Delta_A - i\kappa\right] + \eta_A \left[\int dx \hat{\psi}_\downarrow^\dagger \cos\left(k_0 x\right) \hat{\psi}_\uparrow + h.c.\right]$$

- Steady state solution

$$\alpha \approx \frac{\eta_A \int dx \cos\left(k_0 x\right) \left[\langle \hat{\psi}_\downarrow^\dagger \hat{\psi}_\uparrow \rangle + h.c.\right]}{\Delta_A + i\kappa - \xi_A \sum_\sigma \int dx \langle \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma \rangle \cos^2\left(k_0 x\right)}$$

Effective Hamiltonian

OL generated by Longitudinal Pumping

Cavity Field

Effective Zeeman Splitting

$$\hat{H} = \sum_{\sigma} \int dx \hat{\psi}^{\dagger}_{\sigma} \left[\frac{p_x^2}{2m} + \left(V_0 + \xi_A \hat{a}^{\dagger} \hat{a} \right) \cos^2 \left(k_0 x \right) + \xi_{\sigma} m_z \right] \hat{\psi}_{\sigma}$$

$$+ \eta_A (\alpha^* + \alpha) \left[\int dx \hat{\psi}^{\dagger}_{\uparrow} \cos(k_0 x) \hat{\psi}_{\downarrow} + \mathrm{H.C.} \right]$$

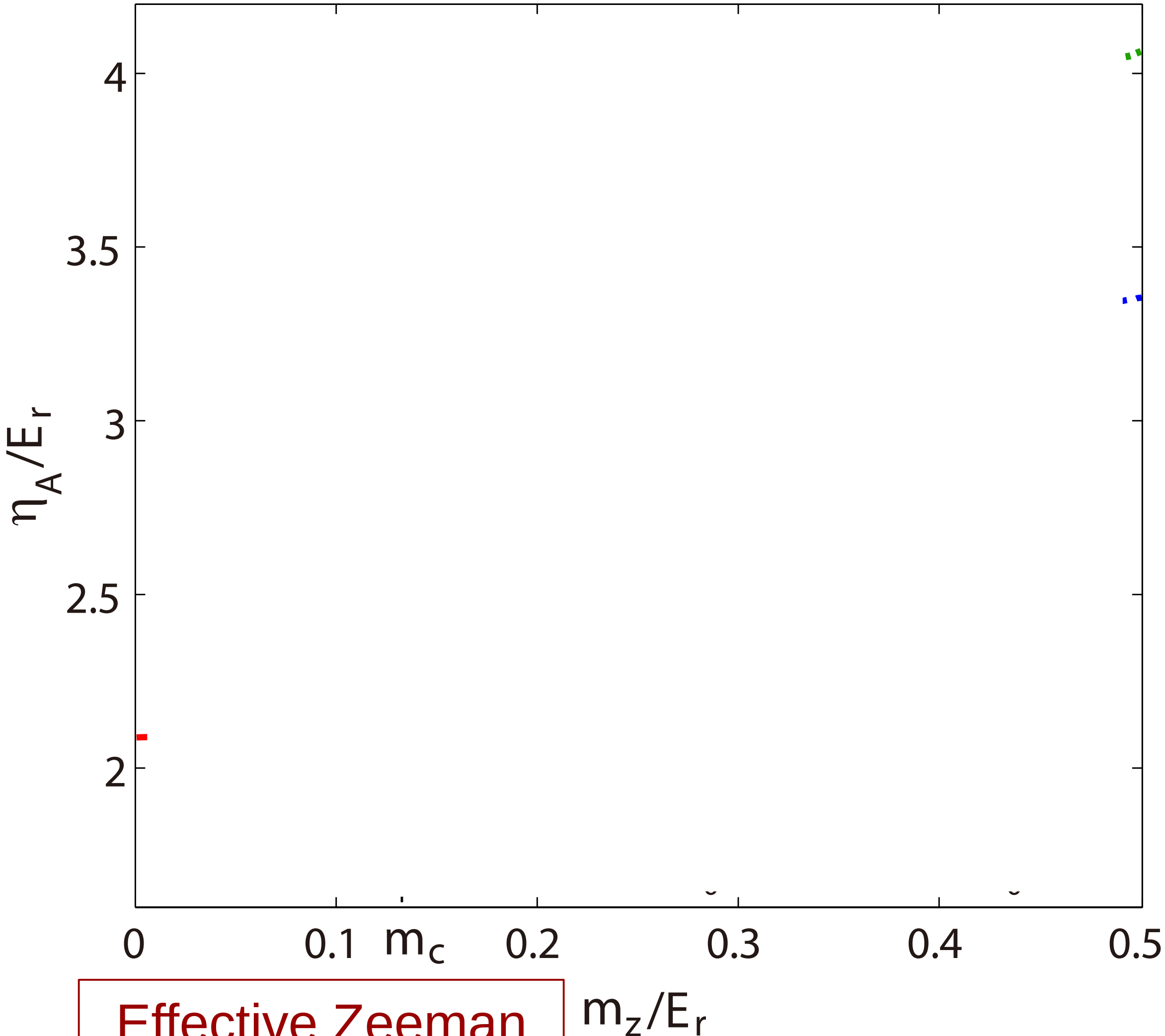
Transverse Pumping

- Solve with

$$\alpha \approx \frac{\eta_A \int dx \cos\left(k_0 x\right) \left[\langle \hat{\psi}^{\dagger}_{\downarrow} \hat{\psi}_{\uparrow} \rangle + h.c. \right]}{\Delta_A + i\kappa - \xi_A \sum_{\sigma} \int dx \langle \hat{\psi}^{\dagger}_{\sigma} \hat{\psi}_{\sigma} \rangle \cos^2 \left(k_0 x \right)}$$

Phase diagram

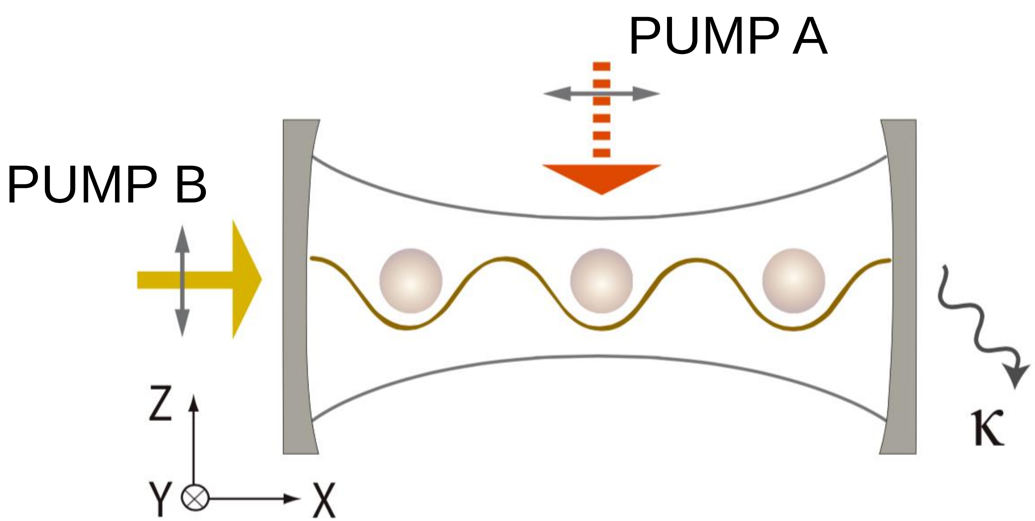
Transverse Pumping

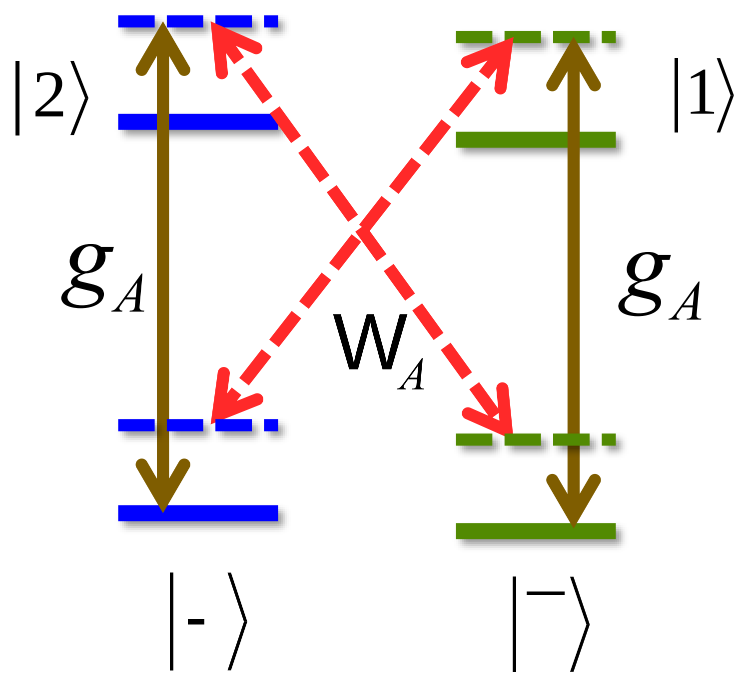


Effective Zeeman Splitting

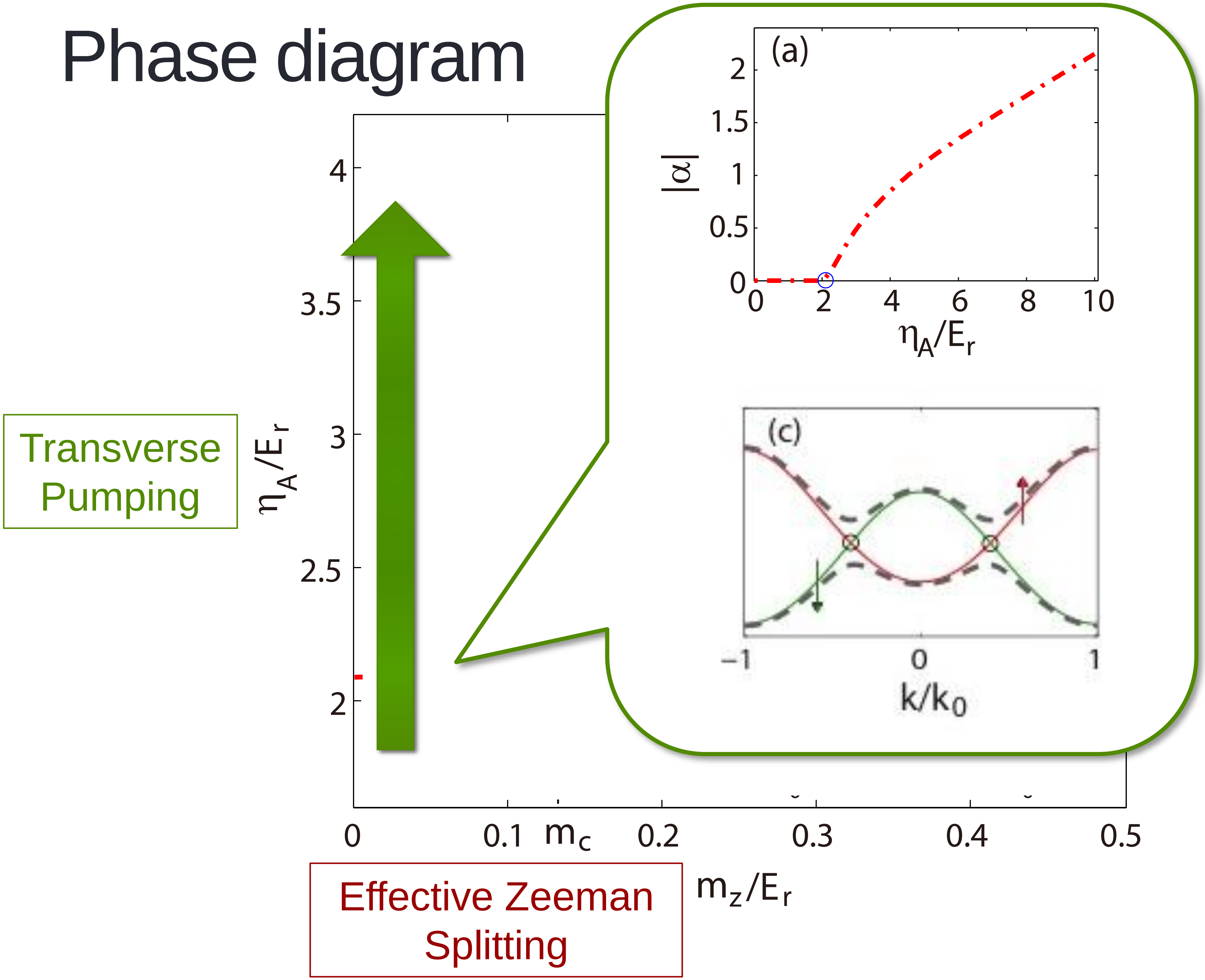
System

- A quasi-1D two-component Fermi gas in a cavity

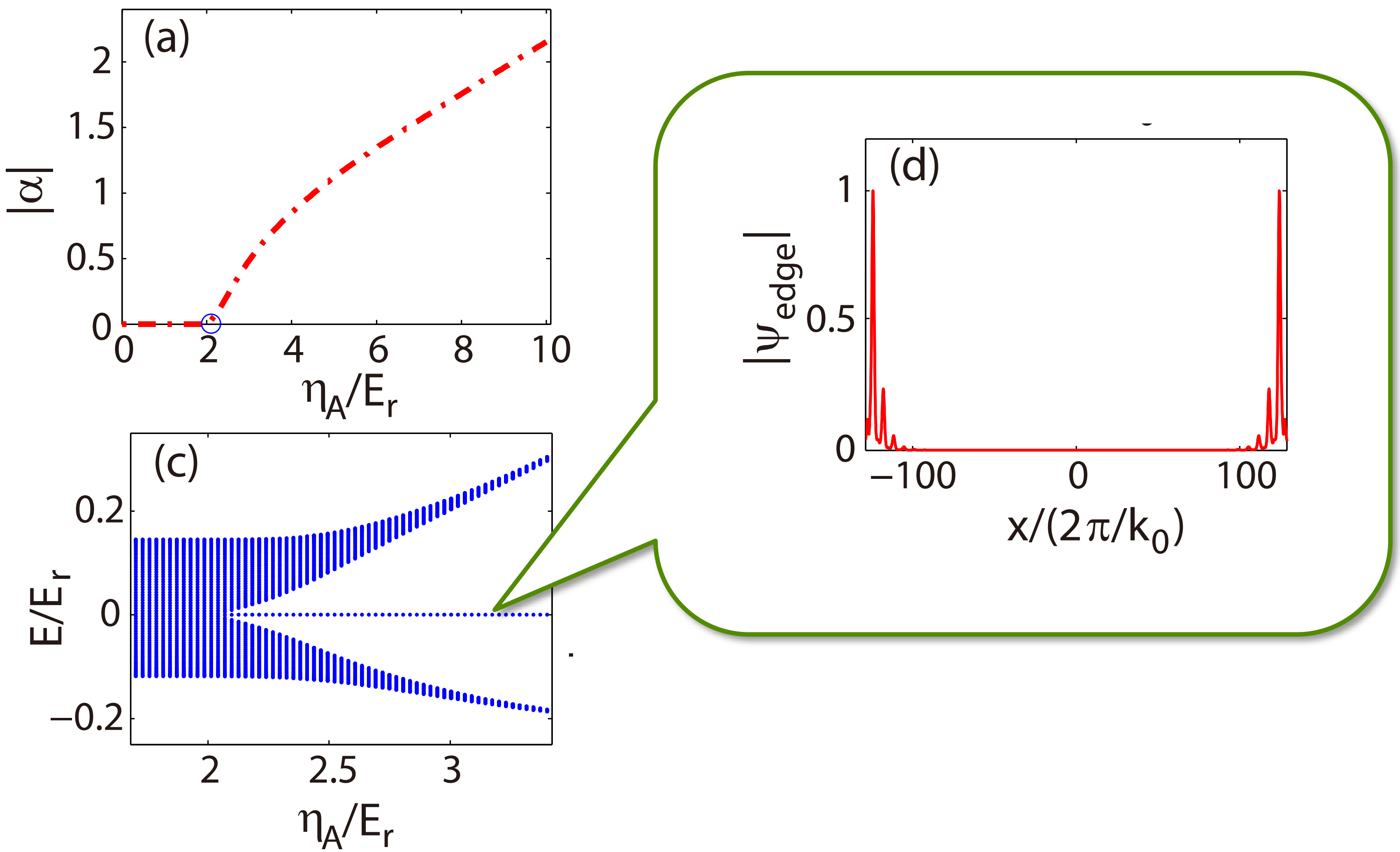




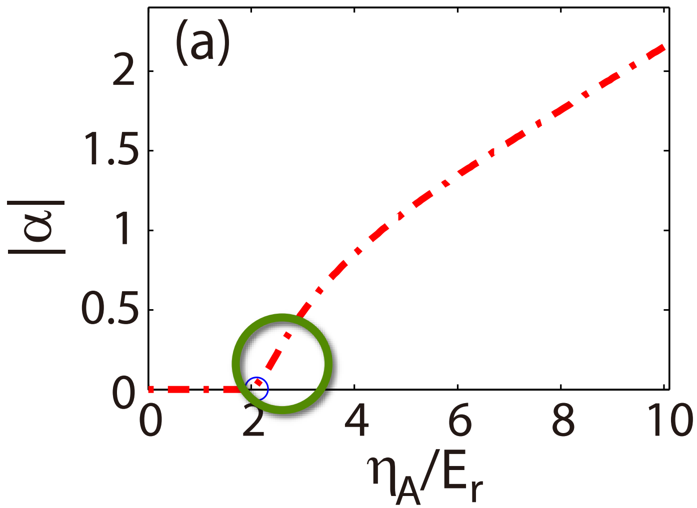
Phase diagram



Topological properties



Topological properties



$$\hat{H}_{TI} = -\sum_{\langle i,j\rangle,\sigma} t_s^{1,1}(i,j)\hat{\psi}^\dagger_{i\sigma}\hat{\psi}_{j\sigma} + \sum_{\langle i,j\rangle}\left(t_{so}^{1,1}(i,j)\hat{\psi}^\dagger_{i\uparrow}\hat{\psi}_{j\downarrow} + \text{H.C.}\right) + m_z\sum_{i,\sigma}\xi_\sigma\hat{\psi}^\dagger_{i\sigma}\hat{\psi}_{i\sigma},$$

$$t_s^{n,n'}(i,j) = \int dx\phi^*_{ni}(x)\left[\frac{\hat{p}_x^2}{2m} + \left(V_0 + \xi_A|\alpha|^2\right)\cos^2(k_0x)\right]\phi_{n'j}(x),$$

$$t_{so}^{n,n'}(i,j) = \eta_A(\alpha^* + \alpha)\int dx\phi^*_{ni}(x)\cos(k_0x)\phi_{n'j}(x),$$

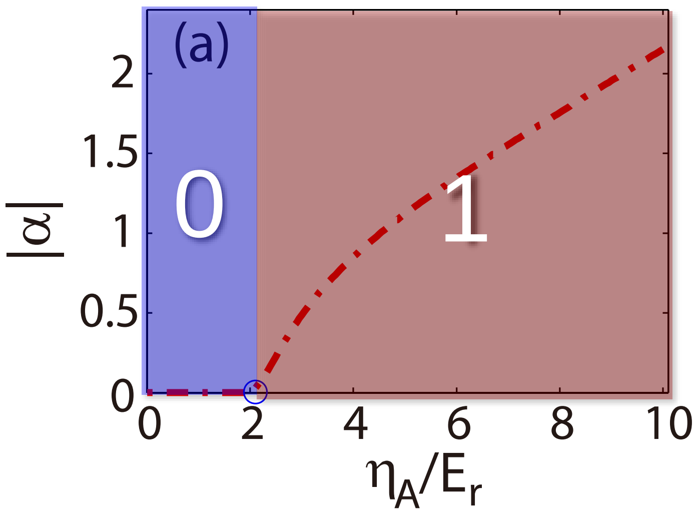
Topological properties

$$\hat{H}_{TI} = \sum_{k\in BZ} \begin{pmatrix} \hat{\psi}^\dagger_{k\uparrow} & \hat{\psi}^\dagger_{k\downarrow} \end{pmatrix} \left[h_y(k)\,\sigma_y + h_z(k)\,\sigma_z\right] \begin{pmatrix} \hat{\psi}_{k\uparrow} \\ \hat{\psi}_{k\downarrow} \end{pmatrix},$$

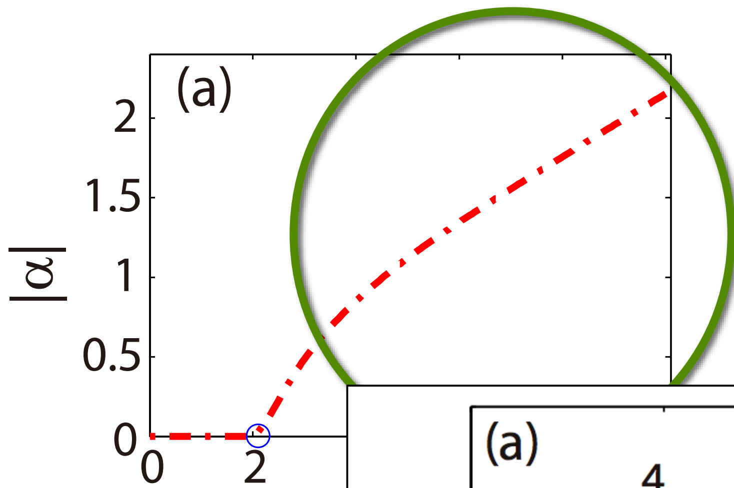
$$h_y(k) = 2t_{so}\sin(ka)$$

$$h_z(k) = m_z - 2t_s\cos(ka)$$

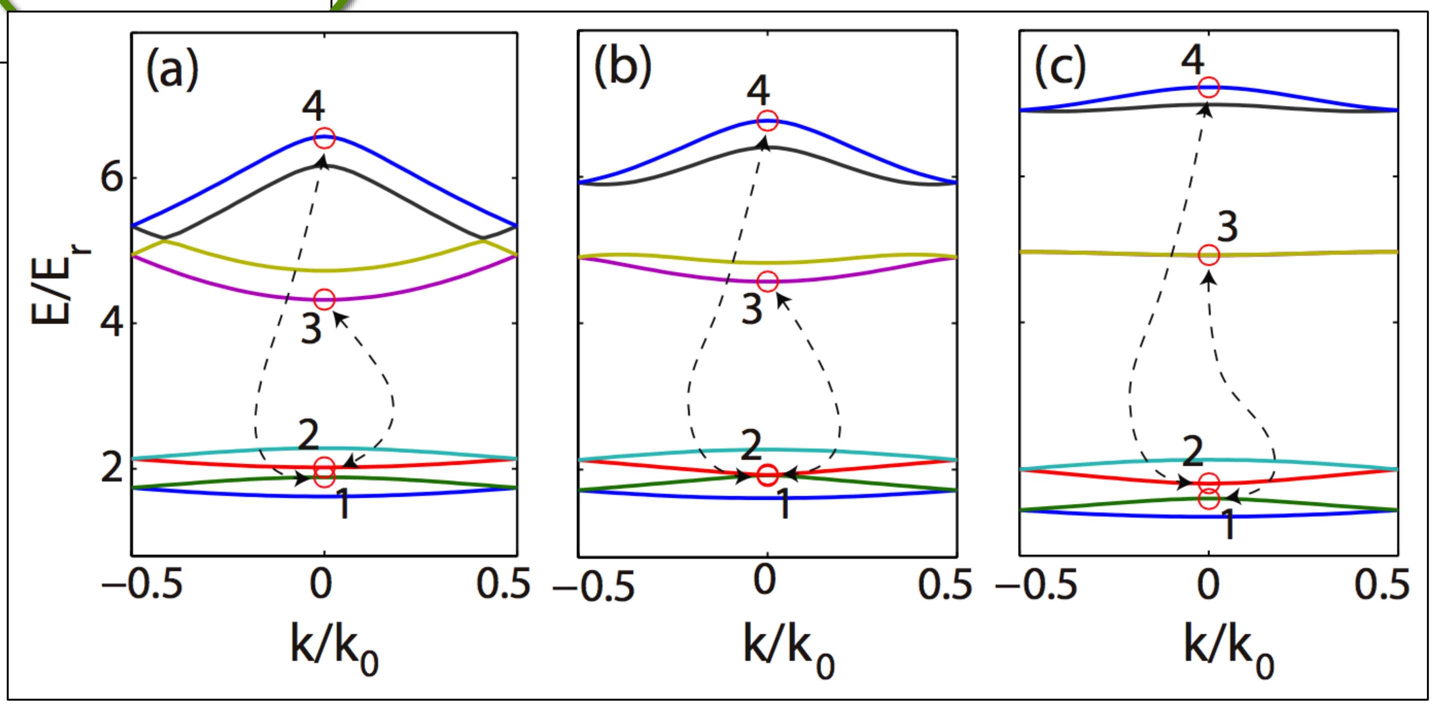
$$\mathcal{W} = \sum_{\nu,\nu'=y,z} \oint \frac{dk}{4\pi} \epsilon_{\nu,\nu'} \hat{h}_\nu^{-1}(k)\,\partial_k \hat{h}_{\nu'}(k),$$



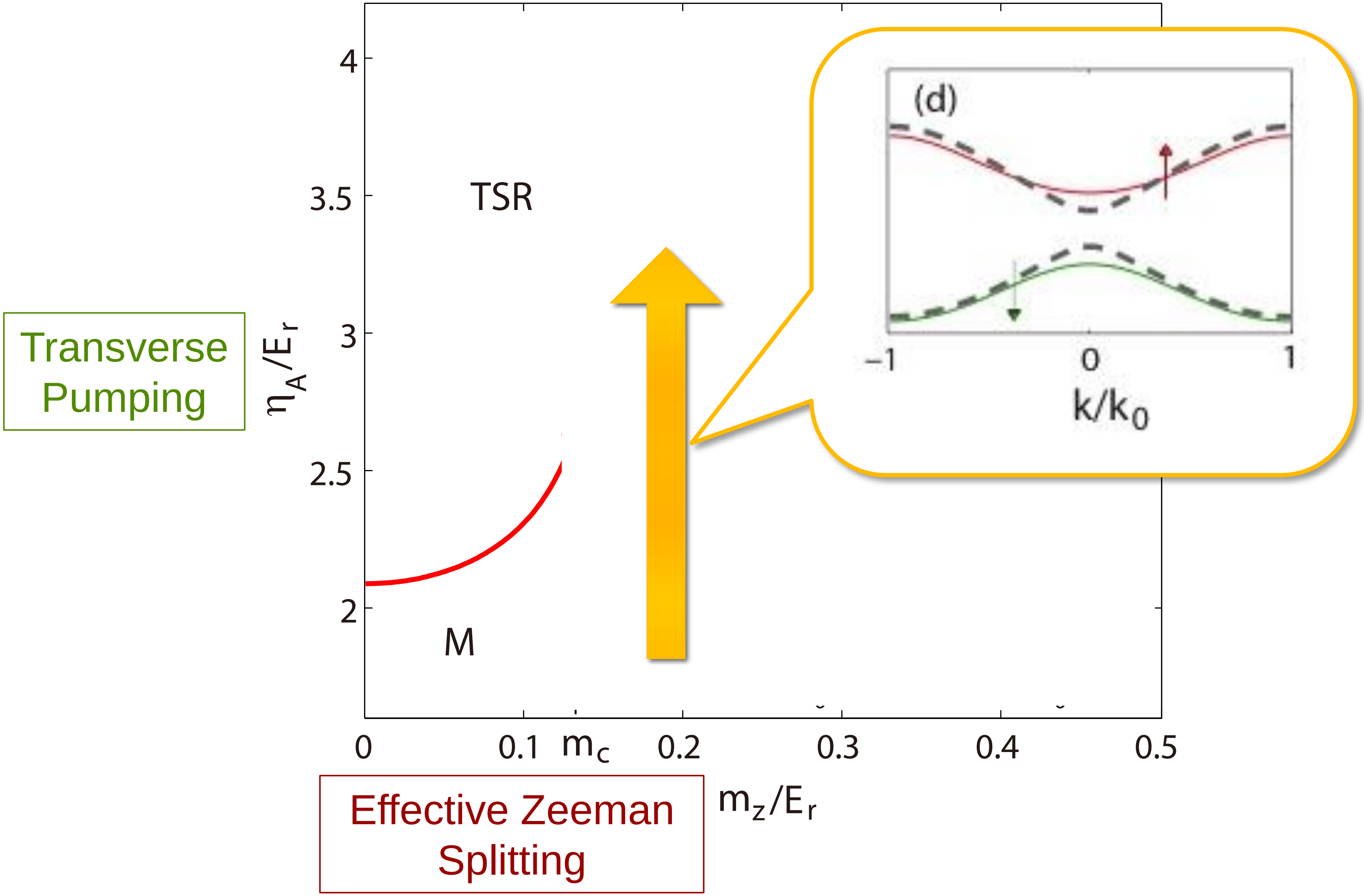
Topological properties



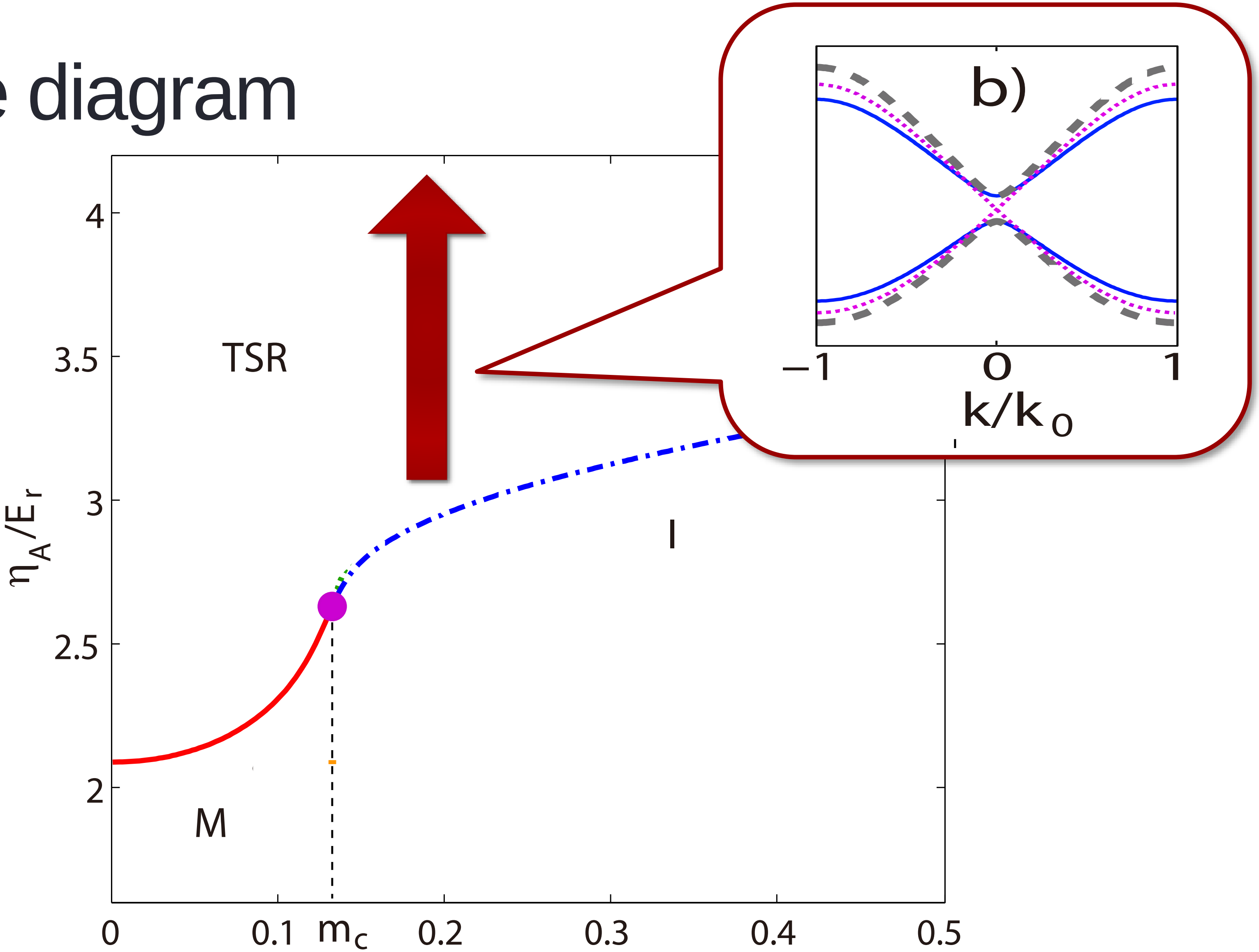
See also, Zhou, Cui., arXiv:1507.01341



Phase diagram



Phase diagram

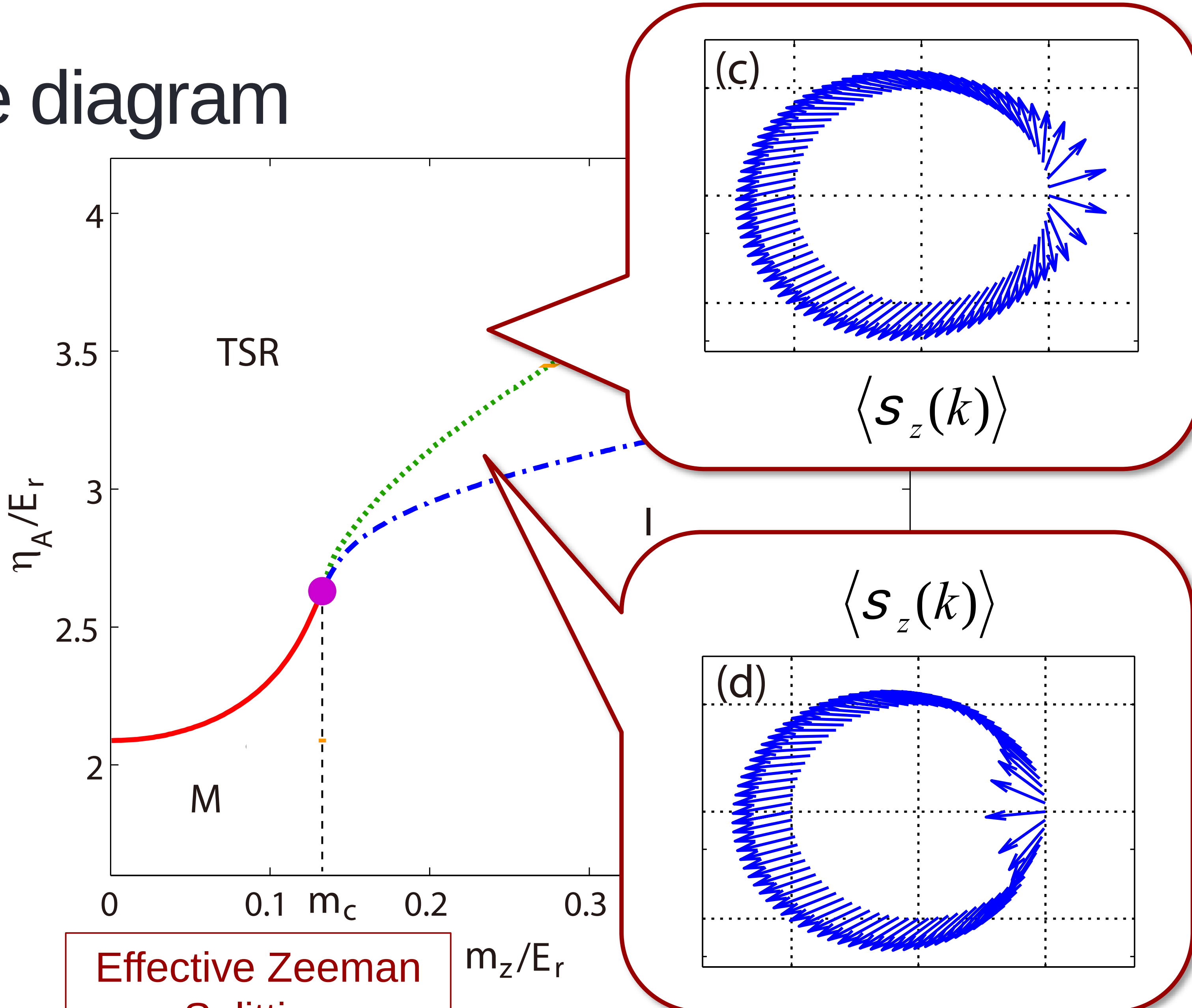


Transverse Pumping

Effective Zeeman Splitting

Phase diagram

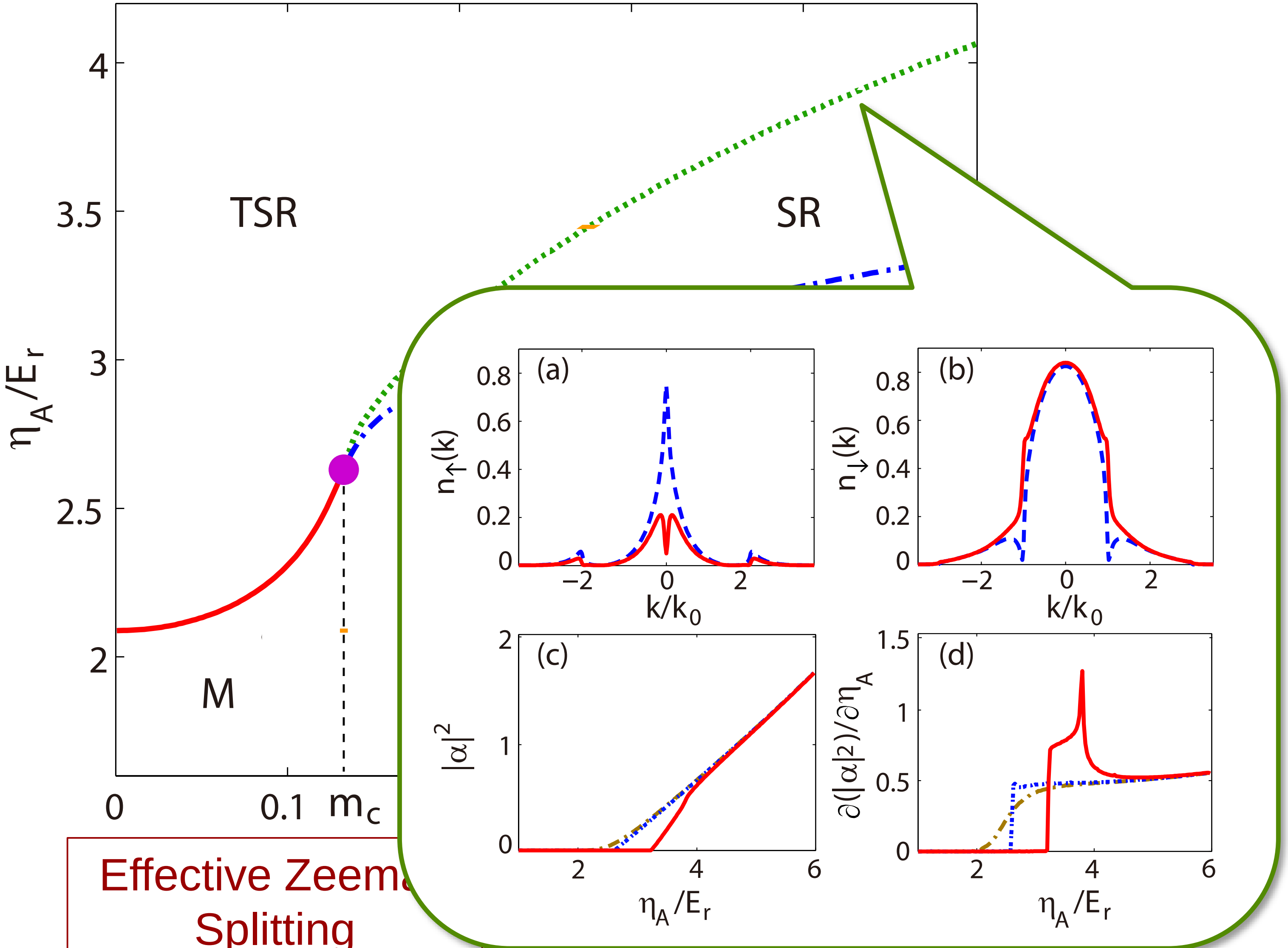
Transverse Pumping



Effective Zeeman Splitting

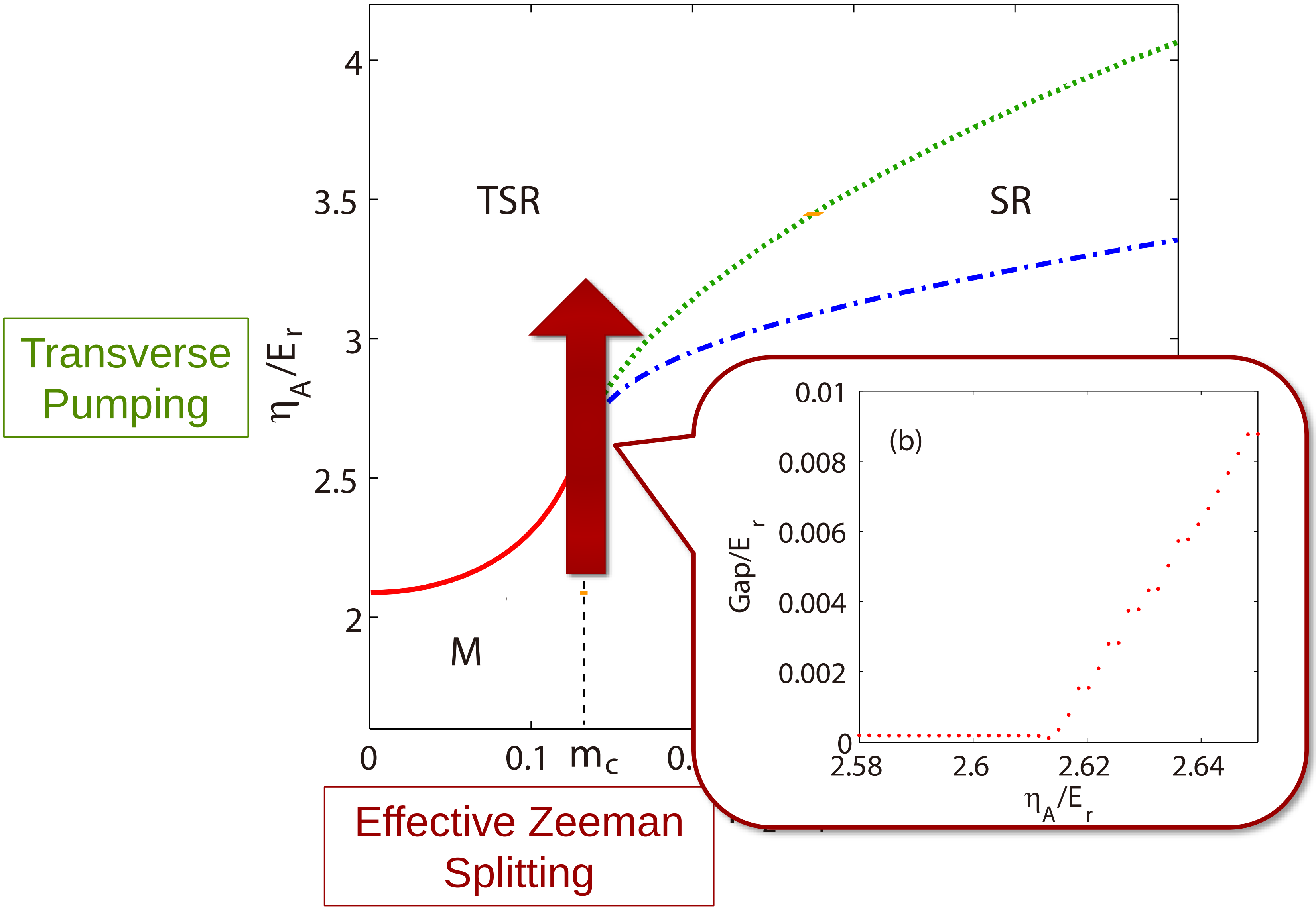
Observables

Transverse Pumping

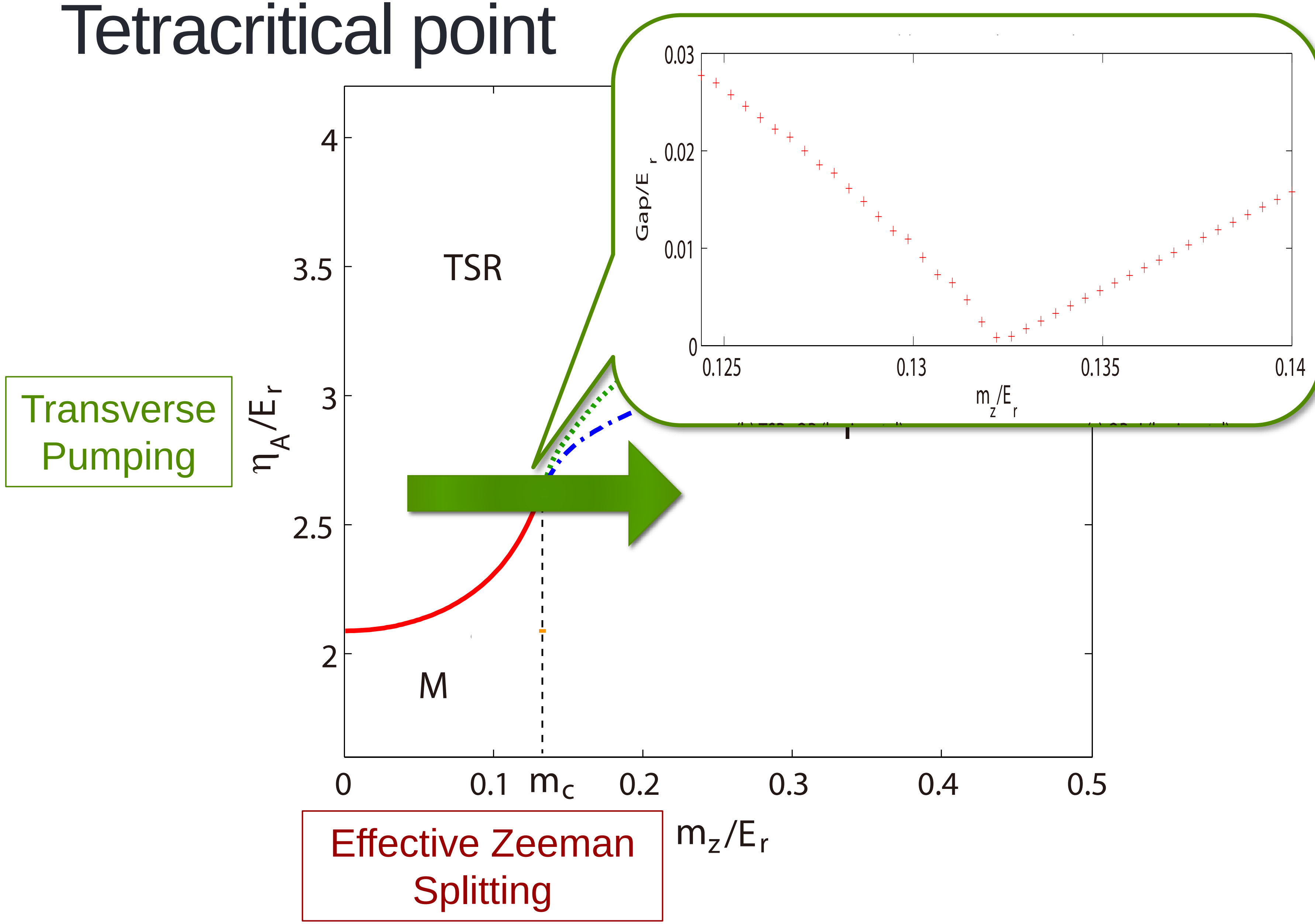


Effective Zeeman Splitting

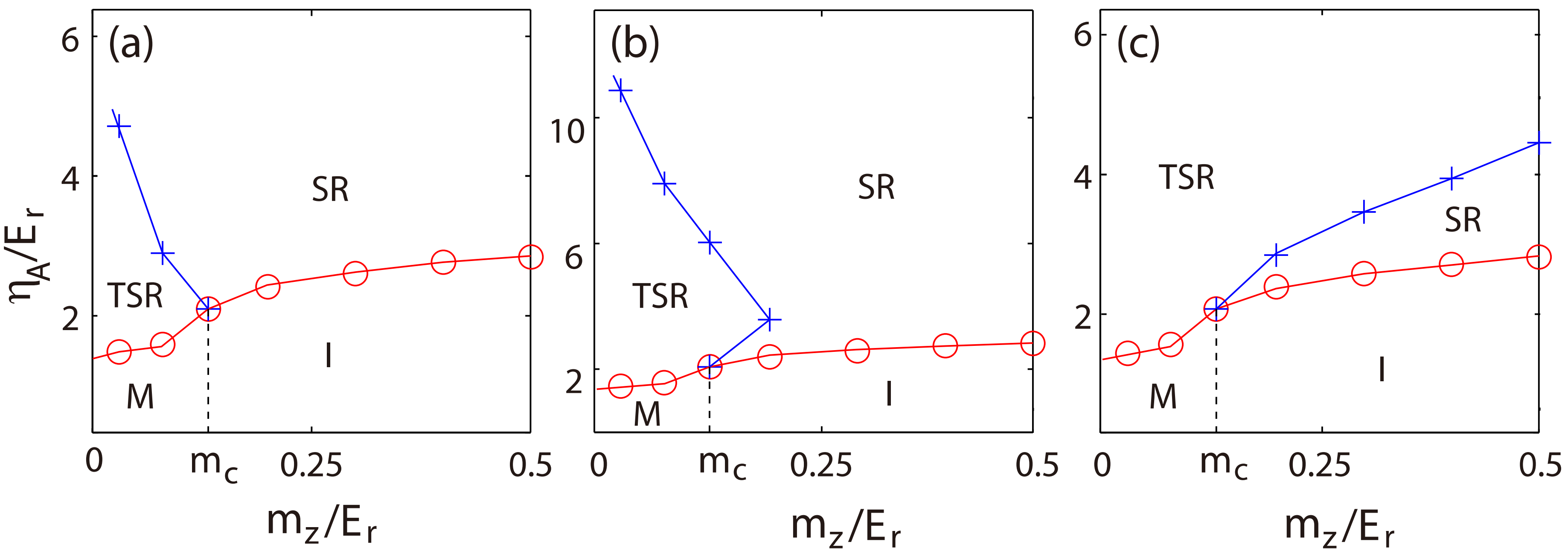
Tetracritical point



Tetracritical point



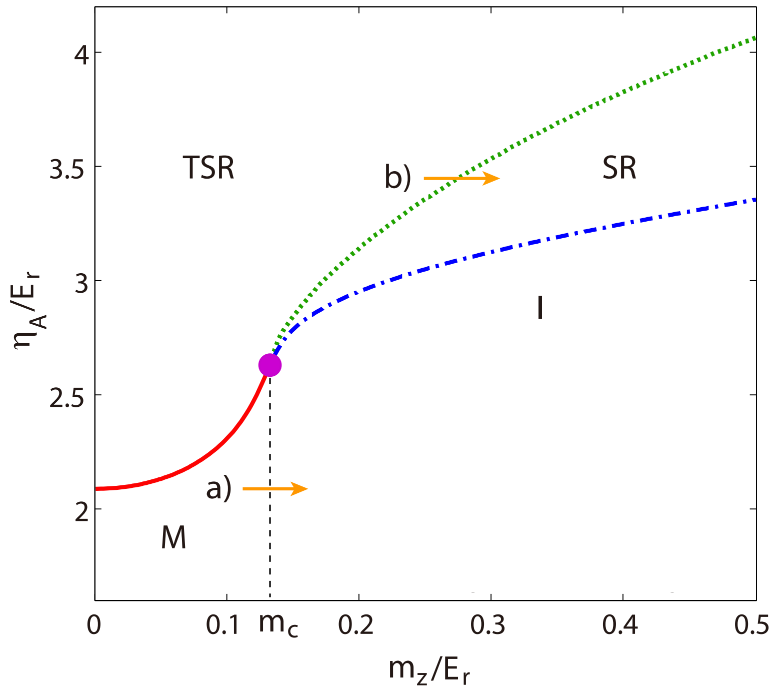
Higher bands effects



1-band approximation 2-band approximation 3-band approximation

Summary

- Superradiance of a quasi-1D non-interacting Fermi gas in a cavity
- Synthetic SOC realized by Raman scheme with driving and cavity laser fields
- Topological SR state



PRL 115, 045303 (2015)
Prog. Phys. 37(4), 125 (2017)
Front. Phys. 13, 136701 (2018)