第七届谱方法及相关应用进展研讨会

Time: June 8-9, 2019

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Regularized weighted least squares approximation by orthogonal polynomials

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Abstract: We consider polynomial approximation over the interval [-1,1] by a class of regularized weighted discrete least squares methods with ℓ_2-regularization and ℓ_1-regularization terms, respectively. It is merited to choose classical orthogonal polynomials as basis sets of polynomial space with degree at most L. As node sets we use zeros of orthogonal polynomials such as Chebyshev points of the first kind, Legendre points. The number of nodes, say $N+1$, is chosen to ensure $L\leq2N+1$. With the aid of Gauss quadrature, we obtain approximation polynomials of degree L in closed form without solving linear algebra or optimization problem. As a matter of fact, these approximation polynomials can be expressed in the form of barycentric interpolation formula when the interpolation condition is satisfied. We then study the approximation quality of ℓ_2-regularization approximation polynomial, especially on the Lebesgue constant. Moreover, the sparsity of ℓ_1-regularization approximation polynomial, respectively. Finally, we give numerical examples to illustrate these theoretical results and show that well-chosen regularization parameter can provide good performance approximation, with or without contaminated data.

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On the construction of H^2 and $H^2(\curl)$ conforming spectral elements in two dimensions

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Abstract: This talk is oriented for the conforming spectral elements for solving fourth order elliptic equations and quad-curl equations on triangulated and/or quadrilated meshes. We start with the structure exploration of the H^2-conforming piecewise polynomial space on triangular/quadrilateral mesh; the interior, edge and vertex modes of the H^2-conforming basis functions are technically constructed through an affine/bilinear mapping with the help of generalized Jacobi/Koornwinder polynomials. In the sequel, we resort to the contravariant transformation, the de Rham complex and the generalized Jacobi/Koornwinder polynomials to complete the construction of the basis functions of $H^2(\curl)$-conforming spectral elements. Finally, numerical experiments are demonstrated to show the effectiveness and accuracy of our conforming spectral element approximation.

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Asymptotic method of Gauss-Jacobi quadrature

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Abstract：In this paper, we apply Burmann’s theorem to the Jacobi polynomial. Asymptotic expansions of the corresponding nodes and weights of the quadrature are derived and these are further used to design a fast and iteration-free algorithm. Numerical results are provided to illustrate the accuracy of the derived algorithm. This is a joint work with Haiyong Wang.

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Variant of scalar auxiliary variable approaches for gradient flows

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Abstract: In this talk, we will propose and analyze a new class of schemes based on a variant of the scalar auxiliary variable (SAV) approaches for gradient flows. Precisely, we construct more robust first and second order unconditionally stable schemes by introducing a new defined auxiliary variable to deal with nonlinear terms in gradient flows. The new approach consists in splitting the gradient flow into decoupled linear systems with constant coefficients, which can be solved using existing fast solvers for the Poisson equation. This approach can be regarded as an extension of the SAV method; see, e.g., [Shen et al., J. Comput. Phys. 2018], in the sense that the new approach comes to be the conventional SAV method when $\alpha = 0$ and removes the boundedness assumption on $\int F(\phi)dx$ required by the SAV. The new approach only requires that the total free energy or a part of it is bounded from below, which is more

realistic in physically meaningful models. The unconditional stability is established, showing that the efficiency of the new approach is less restricted to particular forms of the nonlinear terms. A series of numerical experiments is carried out to verify the theoretical claims and illustrate the efficiency of our method.

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Optimal representations of polynomials and global smooth functions by deep neural networks with rectified power units

李波
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Abstract: Recently, deep neural networks with rectified linear units (ReLU) get very popular due to its universal representation power and successful applications. In this talk, we show that deep networks with rectified power units (RePU) can give better approximations for smooth functions by an optimal explicit construction of neural networks with nite number of RePU units to accurately represent any univariate and multivariate polynomials without any approximation error. Thus the error of best polynomial approximation is an uppper bound of the approximation error of best RePU network approximation. In particular, for a class of high dimensional functions, the sparse grid and hyperbolic cross spectral approximation are used to construct an upper bound of the approximation error of RePU networks. Our constructive proofs show clearly the connections between spectral methods and deep neural networks. Numerical experiments are carried out to check the error bounds and the advantages of the proposed neural networks.

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Polynomial approximation of singular functions in some new fractional Sobolev spaces

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Abstract: In this talk, we introduce a new theoretical framework built upon fractional Sobolev-type spaces involving Riemann-Liouville fractional integrals/derivatives for optimal error estimates of orthogonal polynomial approximations to functions with limited regularity. It naturally arises from exact representations of orthogonal polynomial expansion coefficients. Here, the essential pieces of the puzzle for the error analysis include (i) fractional integration by parts (under the weakest possible conditions), and (ii) generalised Gegenbauer functions of fractional degree (GGF-Fs): a new family of special functions with notable fractional calculus properties. (based on joint works with Li-Lian Wang and Huiyuan Li).

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Efficient space-time spectral method for fractional PDEs

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Abstract: In this talk, we develop an efficient space-time spectral method for nonlinear parabolic PDEs with spectral fractional Laplacian on bounded domains with Dirichlet, the Neumann, and the mixed boundary conditions. The key idea is to construct discrete eigenfunctions (also referred to Fourier-like basis) of spectral fractional Laplacian by using the Fourierization method. We provide a rigorous error analysis for the proposed method, as well as ample numerical results to show its effectiveness

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% **An efficient iterative method for solving multiple scattering in inhomogeneous media**

汪波
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Abstract： An efficient iterative method is proposed for solving multiple scattering problem in inhomogeneous media. The key idea is to enclose the inhomogeneity of the media by well separated artificial boundaries and then apply purely outgoing wave decomposition for the scattering field outside the enclosed region. As a result, the original multiple scattering problem is decomposed into a finite number of single scattering problems, where each of them communicates with the other scattering problems only through its surrounding artificial boundary. Accordingly, they can be solved in a parallel manner at each iteration. This framework enjoys a great flexibility in using different combinations of iterative algorithms and single scattering problem solvers. The spectral element method seamlessly integrated with the nonreflecting boundary condition and the GMRES iteration is advocated and implemented in this work. The convergence of the proposed method is proved by using the compactness of involved integral operators.

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Analysis of Fourier approximations to periodic functions with fractional-order singularities

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Abstract: It is well known that Fourier series is a powerful approach for approximating periodic functions. However, existing approximation results on Fourier approximations to differentiable periodic functions are suboptimal for some differentiable periodic functions. In this talk, I will show how to derive optimal estimates for the Fourier coefficients of periodic functions with a fractional-order singularity. The basic idea is inspired by the recent work by Liu, Wang and Li for the estimate of Chebyshev coefficients.

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Fast spectral methods for PDEs with integral fractional Laplacian in multi-dimensional unbounded domains

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Abstract: PDE with integral fractional Laplacian is a powerful in modelling anomalous diffusion and nonlocal interactions, but its numerical solution can be very difficult especially in multiple dimensions. In fact, many of such nonlocal models are more physically motivated and naturally set in unbounded domains. In this talk, we shall present a superfast spectral-Galerkin method with two critical components (i) based on the Dunford-Taylor formulation of fractional Laplacian operator, and (ii) using Fourier-like mapped Chebyshev functions as basis. We shall also report some of our recent attempts for integral fractional Laplacian in bounded domains, which are deemed even more notoriously difficult in effective numerical discretisations. Along this line, we work with the formulation associated with the Fouririer transformations, and derive a number of useful analytical formulas which are useful for the algorithm development.

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The numerical study of regularized barycentric interpolation

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Abstract: Regularized barycentric interpolation formulae were introduced in our previous paper, which involve ℓ_2 and ℓ_1 regularization terms, respectively. Naturally, regularized barycentric interpolation formulae could be applied to recover contaminated functions with well distributed interpolation nodes. In this talk, we introduce modified regularized Lagrange interpolation formula based on the so-called first barycentric interpolation, given by C. Jacobi in 1825. Then we focus on the numerical stability of these regularized interpolation formulae in terms of backward and forward stability. We also involve the stability with respect to extrapolation, illustrating regularized modified Lagrange interpolation is better than regularized barycentic interpolation in extrapolation. Moreover, we employ Chebyshev points (1st and 2nd kind, respectively) and Legendre points as interpolation nodes to test numerical stability.

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Interpolation and expansion on orthogonal polynomials

向淑晃
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Abstract: The convergence rates on polynomial interpolation in most cases are estimated by Lebesgue constants. These estimates may be overestimated for some special points of sets for functions of limited regularities. In this talk, new formulas on the convergence rates are considered. Moreover, new and optimal asymptotics on the coefficients of functions of limited regularity expanded in forms of Jacobi and Gegenbauer polynomial series are presented. All of these asymptotic analysis are optimal. Numerical examples illustrate the perfect coincidence with the estimates.

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Computing convolutions, the story so far

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Abstract: Convolution is found ubiquitously dense in mathematics and engineering. In this talk, we'll review classic results in literature on convolution quadrature before moving on to the most recent development of powerful numerical methods for computing convolution integrals. Based on the spectral approximation of convolution operators via classic orthogonal polynomials or Fourier extensions, we'll arrive at fast and spectrally-accurate algorithms which make the calculation of convolution integrals possible in the sense of "computing with functions" and these new methods are believed to lay the very foundation of the first spectral methods for convolution integral equations.

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Numerical solutions of two kinds of time-fractional Maxwell's equations in dispersive media

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Abstract: Over the years, dispersion modelling has become one of the key topics in computational electrodynamics largely for the reason that many materials have frequency dependent properties, including biological tissues, soils, polymers. In this talk, we will give numerical solutions of Maxwell's equations in two kinds of dispersive media, i.e., Davidson-Cole and Havriliak-Negami medium, which involve two kinds of non-standard time-fractional integral operators. Based on the properties of the non-standard time-fractional integral operators, direct and fast algorithms are proposed to approximate them. Then, the fully-discrete schemes of time-fractional Maxwell's equations are derived. Some theoretic results of the continuous and discrete problems are also given. Finally, some numerical examples are given to validate the theoretic results.

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An hp-version of the C^0-continuous Galerkin time-stepping method for nonlinear second order initial value problems

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Abstract: In this talk we shall present an hp-version of the C^0-continuous Galerkin time-stepping method for nonlinear second order initial value problems. We combine the continuous and discontinuous Galerkin time-stepping methodologies to obtain natural discretization of the second time derivative. We derive a-priori error estimate in the H^1-norm that is fully explicit with respect to the local time steps, the local polynomial degrees, and the local regularity of the exact solution. In particular, we show that the C^0-continuous Galerkin scheme based on geometrically refined time steps and on linearly increasing approximation orders achieves exponential rates of convergence for solutions with start-up singularities. The theoretical results are illustrated by some numerical experiments. This talk is based on joint work with Yichen Wei.

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Efficient energy stable schemes for time-fractional phase-field systems

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Abstract: For the time-fractional phase field models, the corresponding energy dissipation law has not been settled on both the continuous level and the discrete level. In this work, we shall address this open issue. More precisely, we prove that the time-fractional phase field models indeed admit an energy dissipation law of an integral type. In the discrete level, we propose a class of finite difference schemes that can inherit the theoretical energy stability. Our discussion covers the time-fractional gradient systems including the time-fractional Allen-Cahn equation, the time-fractional Cahn-Hilliard equation and the time-fractional molecular beam epitaxy models. Moreover, a numerical study of the coarsening rate of random initial states depending on the fractional parameter α reveals that there are several coarsening stages for both time-fractional Cahn-Hilliard equation and time- fractional molecular beam epitaxy model, while there exists a $-\alpha/3$ power law coarsening stage. This talk is based on joint works with: Tao Tang (SUSTC) and Tao Zhou (AMSS).

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Two types of generalizations of prolate spheroidal wave functions (PSWFs)

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Abstract: In this talk, we first introduce the prolate spheroidal wave functions (PSWFs) of real order $\alpha > -1$ on the unit ball in arbitrary dimension, termed as ball PSWFs. They are eigenfunctions of both a weighted concentration integral operator, and a Sturm-Liouville differential operator. Different from existing works on multi-dimensional PSWFs, the ball PSWFs are defined as a generalisation of orthogonal ball polynomials in primitive variables with a tuning parameter $c > 0$, through a "perturbation" of the Sturm-Liouville equation of the ball polynomials. In second part, with the divergence Free Curl-Orthogonal Polynomials, we introduce the vectorial ball prolate spheroidal wave functions (PSWFs) and their properties.

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Optimal error estimate of spectral methods with quasi-optimal complexity for one dimensional fractional advection diffusion equations

张中强
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Abstract:

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Multi-domain spectral collocation method for nonlinear variable-order fractional differential equations

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Abstract: Spectral and spectral element methods using Galerkin type formulations are efficient for solving linear fractional PDEs (FPDEs) of constant order but are not efficient in solving nonlinear FPDEs and cannot handle FPDEs with variable-order. In this paper, we present a multi-domain spectral collocation method that addresses these limitations. We consider FPDEs in the Riemann–Liouville sense, and employ Jacobi Lagrangian interpolants to represent the solution in each element. We provide variable-order differentiation formulas, which can be computed efficiently for the multi-domain discretization taking into account the nonlocal interactions. We enforce the interface continuity conditions by matching the solution values at the element boundaries via the Lagrangian interpolants, and in addition we minimize the jump in (integer) fluxes using a penalty method. We analyze numerically the effect of the penalty parameter on the condition number of the global differentiation matrix and on the stability and convergence of the penalty collocation scheme. We demonstrate the effectiveness of the new method for the fractional Helmholtz equation of constant and variable-order using $h - p$ refinement for different values of the penalty parameter. We also solve the fractional Burgers equation with constant and variable-order and compare with solutions obtained with a single domain spectral collocation method.

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Reduced-Order Modelling for the Allen-Cahn equation based on scalar auxiliary variable approaches

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Abstract: We study the reduced-order modelling for Allen-Cahn equation. First, a collection of phase field data, i.e., an ensemble of snapshots of at some time instances is obtained from numerical simulation using a time-space discretization. The full discretization makes use of a temporal scheme based on the scalar auxiliary variable approach and a spatial spectral Galerkin method. It

is shown that the time stepping scheme is unconditionally stable. Then a reduced order method is developed using by proper orthogonal decomposition (POD) and discrete empirical interpolation method (DEIM). It is well-known that the Allen-Cahn equations have a nonlinear stability property, i.e., the free-energy functional decreases with respect to time. Our numerical experiments show that the discretized Allen-Cahn system resulting from the POD-DEIM method inherits this favorable property by using the scalar auxiliary variable approach. A few numerical results are presented to illustrate the performance of the proposed reduced order method. In particular, the numerical results show that the computational efficiency is significantly enhanced as compared to directly solving the full order system.

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