International Conference on Eigenvalue Problems and Related Topics

Conference Manual

May 8-9, 2021

北京计算科学研究中心
BEIJING COMPUTATIONAL SCIENCE RESEARCH CENTER

北京工業大學
BEIJING UNIVERSITY OF TECHNOLOGY

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International Conference on Eigenvalue Problems and Related Topics

Brief Introduction

Eigenvalue problems arise in many scientific and engineering applications. The topics of the conference involve rapidly developments at the frontiers on today's research related to eigenvalue problems. The conference focuses on the following topics at the forefront of research: 1) numerical approximation of PDE eigenvalue problems; 2) eigensolvers of large, sparse, non-Hermitian matrices; 3) eigenvalue problems arising from physics, big data sciences, optimization, etc. The conference will feature plenary talks given by international experts on the aforementioned topics and mini-symposium talks mainly from junior researchers. We expect to have a sequence of inspiring talks on various problems.

Aim

The main goal of the conference is to bring together an international collection of established specialists as well as junior researchers. This type of scientific gathering and interaction can have long-lasting impact on junior researchers and graduate students. A significant broader impact will be to bring young researchers into the field, and establish new connections between disparate groups. In addition, there will be a significant attendance of graduate students. The workshop is a great opportunity for them to access the cutting-edge research, find interesting research topics, learn from world leading researchers, and build the network/collaboration for their future academic careers.

Speakers

Plenary Speakers:

Daniele Boffi, King Abdullah University of Science and Technology

Peter Monk, University of Delaware

Zhaojun Bai, University of California, Davis

Aihui Zhou, Chinese Academy of Sciences

Invited Speakers:

Jing An, Beijing Computational Science Research Center

Huajie Chen, Beijing Normal University

Jintao Cui, Hong Kong Polytechnic University

Bo Gong, Beijing University of Technology

Hailong Guo, University of Melbourne

Jiayu Han, Guizhou Normal University

Xia Ji, Chinese Academy of Sciences

Hengguang Li, Wayne State University

Huiyuan Li, Chinese Academy of Sciences

Tiexiang Li, Southeast University

Junshan Lin, Auburn University

Xuefeng Liu, Niigata University

Yangfeng Su, Fudan University

Jiguang Sun, Michigan Technological University

Wenqiang Xiao, Beijing Computational Science Research Center

Hehu Xie, Chinese Academy of Sciences

Yan Xu, University of Science and Technology of China

Guojian Yin, The Chinese University of Hong Kong

Fang Zeng, Chongqing University

Hai Zhang, The Hong Kong University of Science and Technology

Conference Program

Date	Time	Content
Saturday May 8	**8:30-8:40**	**Opening Ceremony**
	8:40-10:40	**Chairman: Zhaojun Bai**
	8:40-9:40	Peter Monk: Eigenvalues as target signatures in inverse scattering
	9:40-10:10	Hengguang Li: A C^0 finite element method for the biharmonic problem with the Navier boundary condition
	10:10-10:40	Yan Xu: A parallel eigensolver for photonic crystal discretized by edge finite elements
	10:40-10:50	**Tea Break**
	10:50-11:50	**Chairman: Yan Xu**
	10:50-11:20	Huiyuan Li: On the Laplacian Eigenvalues on Tetrahedra
	11:20-11:50	Xuefeng Liu: Rigorous eigenvalue estimation to the Stokes equation and its application to solution verification for Navier-Stokes equation
Saturday May 8	**13:30-15:00**	**Chairman: Zhimin Zhang**
	13:30-14:30	Daniele Boffi: On the spectrum of operators related to least-squares finite element methods
	14:30-15:00	Jintao Cui: Discontinuous Galerkin methods for fourth order variational inequality and related problems
	15:00-15:30	Hehu Xie: The augmented subspace method for eigenvalue problems
	15:30-15:40	**Tea Break**
	15:40-17:40	**Chairman: Hehu Xie**
	15:40-16:10	Huajie Chen: Plane wave methods for quantum eigenvalue problems of incommensurate systems
	16:10-16:40	Jiayu Han: Error estimates for H(curl2) conforming element of Maxwell's transmission eigenvalue problem using fixed-point approach
	16:40-17:10	Xia Ji: A new proof of the C0IP method for the biharmonic eigenvalue problem
	17:10-17:40	Hai Zhang: Mathematical Studies of Extraordinary Light Transmission Through Subwavelength Holes in Metallic Slabs
Sunday May 9	**8:40-10:40**	**Chairman: Jiguang Sun**
	8:40-9:40	Zhaojun Bai: Thick-Restart Lanczos with explicit external deflation for computing many eigenpairs, and its communication-avoiding variant
	9:40-10:40	Aihui Zhou: Eigenfunction behaviors and adaptive finite element approximations of nonlinear eigenvalue problems

	10:40-10:50	**Tea Break**
	10:50-11:50	**Chairman: Aihui Zhou**
	10:50-11:20	Junshan Lin: Resonances in nano-holes and extraordinary optical transmission
	11:20-11:50	Jiguang Sun: Finite element/holomorphic operator value function approach for nonlinear eigenvalue problems
Sunday May 9	**13:30-15:30**	**Chairman: Jilu Wang**
	13:30-14:00	Yangfeng Su: 2DEVP: theory and algorithms
	14:00-14:30	Hailong Guo: Unfitted computation of edge modes in photonic graphene
	14:30-15:00	Wenqiang Xiao: Finite element calculation of photonic band structures for frequency dependent materials
	15:00-15:30	Jing An: An efficient spectral-Galerkin approximation based on dimension reduction scheme for transmission eigenvalues in polar geometries
	15:30-15:40	**Tea Break**
	15:40-17:40	**Chairman: Qiumei Huang**
	15:40-16:10	Guojian Yin: A contour-integral based method for computing the number of eigenvalues inside a region
	16:10-16:40	Tiexiang Li: Fast Algorithms for Maxwell's Equations for Three-Dimensional Photonic Crystals
	16:40-17:10	Fang Zeng: A spectral projection method for transmission eigenvalues
	17:10-17:40	Bo Gong: Convergence analysis of two finite element methods for the modified Maxwell’s Stekloff eigenvalue problem
	17:40-17:50	**Closing Ceremony**

Title and Abstract Information

Eigenvalues as Target Signatures in Inverse Scattering

Peter Monk

University of Delaware

Abstract: Target signatures are discrete quantities computed from scattering data that might be used to classify scatterers or to detect changes in the properties of scatterers. They we originally suggested as a tool in radar scattering. More recently, Colton, Cakoni and Haddar have shown that transmission eigenvalues can be detected from far field scattering data. This can only be done for non-conducting materials, and requires multifrequency data. These might be a drawbacks, and so instead we have used modified far field operators to approximate Steklov eigenvalues and a new class of modified interior transmission eigenvalues. The resulting target signatures can be computed from single frequency data, and are defined for conducting media. In this talk, I will concentrate on Steklov and modified transmission eigenvalues. In particular I will present new work with F. Cakoni and Yangwen Zhang on using an artificial Steklov eigenvalue problem designed to provide target signatures for a very thin object (a screen) approximated by transmission conditions.

A pseudo-transient method for solving quantum drift-diffusion equations

Weiying Zheng

Chinese Academy of Sciences

Abstract: In this paper, we propose a pseudo-transient method for solving quantum drift-diffusion equations for semiconductor devices. We prove that the approximate problem has a unique solution at each iterative step and the approximate solutions are uniformly bounded in $L^{\infty}(\Omega)\cap H^{1}(\Omega)$. Moreover, assuming that the bias voltage applied to device is small enough, we prove that the approximate solution converges to the exact solution strongly in the H^{1}-norm.Numerical experiments will also be presented to demonstrate the efficiency of the method.

A parallel eigensolver for photonic crystal discretized by edge finite elements

Yan Xu

University of Science and Technology of China

Abstract: A parallel eigensolver for computing band structures of photonic crystals is constructed. The new solver is based on a penalty discrete form derived from modified edge finite element discretization. This form complements the null space of the discrete operator, which make it enough to directly compute the smallest eigenvalues of the matrix eigenvalue problems. We prove that the wanted eigenpairs are free from the influence of the penalty term and can be picked up by a simple reconstruction. We take the locally optimal block preconditioned conjugate gradient (LOBPCG) method as the basic eigensolver and design a matrix-type preconditioner for it. As the algorithm is of fine-grained parallelism, we accelerate it using GPU. Numerical examples are presented to demonstrate the capability and efficiency of the algorithm.

On the Laplacian Eigenvalues on Tetrahedra

Huiyuan Li

Chinese Academy of Sciences

Abstract: In the talk, we explore the Laplacian eigenpairs on some fundamental tetrahedra. Firstly, admissible 3-dimensional root systems together with their corresponding lattices are introduced, whose asymmetric units are exactly the fundamental tetrahedra. Orthogonal families of complex exponentials are established on the primitive cells of the root lattices. Then the Laplacian eigenpairs on these fundamental tetrahedra are obtained from those complex exponentials via Fourier analysis. Some properties on the Laplacian eigenvalues on tetrahedra, such as the fundamental tone, fundamental gap, spectrum partial sum, and spectrum gap distribution, are studied and verified numerically by the Galerkin-spectral method in the sequel.

Rigorous eigenvalue estimation to the Stokes equation and its application to solution verification for Navier-Stokes equation

Xuefeng Liu

Niigata University

Abstract: The eigenvalue problem for the Stokes equation involves the divergence-free condition for the velocity, which causes the difficulty in the error estimation. In our research, two algorithms are developed to process the divergence-free condition in the eigenvalue problems. One is to utilize the Crouzeix-Raviart FEM to construct a local divergence-free space and obtain rigorous lower bounds for the objective eigenvalues. The other algorithm is to apply the extended hypercircle equation to construct an explicit a priori error estimation for the conforming subspace constructed by the Scott-Vogelius FEM. The rigorous eigenvalue estimation for the Stokes equation plays an important role in the solution verification for the Navier-Stokes equation. The obtained rigourous eigenvalues contribute to the first solution verification case to the Navier-Stokes equation in a 3D domain.

On the spectrum of operators related to least-squares finite element methods

Daniele Boffi

Abdullah University of Science and Technology

Abstract: The study of the spectrum of operators arising from the solution of partial differential equations is usually performed for the discretization of the corresponding eigenvalue problems. Besides being an interesting question by itself, it has also several other important applications. For instance, the knowledge of the spectrum gives useful insights for the approximation of transient problems. In this talk we provide a priori and a posteriori error analysis for the spectrum of operators arising from least-squares finite element methods in the case of elliptic problems and linear elasticity.

Discontinuous Galerkin methods for fourth order variational inequality and related problems

Jintao Cui

Hong Kong Polytechnic University

Abstract: In this work we study a family of discontinuous Galerkin methods for the displacement obstacle problem of Kirchhoff plates on convex polyhedral domains, which are characterized as fourth order elliptic variational inequalities of the first kind. We develop a unified approach for DG methods where the weak complementarity form of the variational inequality is used. We prove that the error convergence order for the quadratic method is determined by the geometry of the domain. Under additional regularity assumptions on the solution and contact set, we derive an improved error estimate for the cubic method. We will also discuss possible extensions to other related problems, such as variational eigenvalue inequalities and elliptic optimal control problem with state constraint, etc. Numerical experiments demonstrate the performance of the methods and confirm the theoretical results.

The augmented subspace method for eigenvalue problems

Hehu Xie

Chinese Academy of Sciences

Abstract: In this paper, we introduce a type of augmented subspace method for eigenvalue problems. Based on a low dimensional space, we build au augmented subspace which can transforming solving the eigenvalue problem solving to the solution of linear equations plus low dimensional eigenvalue problems. The way can improve the efficiency for solving eigenvalue problems. In this talk, we will give some new convergence rates and parallel way for solving eigenvalue problems. Some applications will also been introduced.

Plane wave methods for quantum eigenvalue problems of incommensurate systems

Huajie Chen

Beijing Normal University

Abstract: We propose a novel numerical algorithm for computing the electronic structure related eigenvalue problems of incommensurate systems. Unlike the conventional practice that approximates the system by a large commensurate supercell, our algorithm directly discretizes the eigenvalue problems under the framework of a plane wave method. The emerging ergodicity and the interpretation from higher dimensions give rise to many unique features compared to what we have been familiar with in the periodic systems. The numerical results of 1D and 2D quantum eigenvalue problems are presented to show the reliability and efficiency of our algorithm.

Furthermore, the extension of our algorithm to full Kohn-Sham density functional theory calculations is discussed.

Error estimates for H(curl2) conforming element of Maxwell's transmission eigenvalue problem using fixed-point approach

Jiayu Han

Guizhou Normal University

Abstract: This talk is outlined as follows. Based on the fixed-point weak formulation, we prove the error estimates for the curl-curl element methods of the Maxwell's transmission eigenvalue problem. Both real and complex eigenvalues are considered in the error analysis. Under reasonable assumptions, the optimal error estimates for the finite element eigenvalues is proved as well as those in H(curl2)-norm and H(curl2)-norm for finite element eigenfunctions. Numerical experiments are performed to verify the theoretical assumptions as well as confirm our theoretical analysis.

A new proof of the C0IP method for the biharmonic eigenvalue problem

Xia Ji

Beijing Institute of Technology

Abstract: The talk presents a new proof of the C0IP method for the biharmonic eigenvalue problem. We rewrite the problem as the eigenvalue problem of a holomorphic Fredholm operator function of index zero. The convergence for C0IP is proved using the abstract approximation theory for holomorphic operator functions. We employ the spectral indicator method to compute the eigenvalues. Numerical examples are presented to validate the theory.

Mathematical Studies of Extraordinary Light Transmission Through Subwavelength Holes in Metallic Slabs

Hai Zhang

The Hong Kong University of Science and Technology

Abstract: Since the discovery of the extraordinary optical transmission through nanohole arrays inmetalliclms by Ebbesen, a wealth of research has been sparked in the experimental and theoretical investigation of the transmission enhancement in subwavelength nanostructures. In this talk, usingtwo-dimensional periodic slits as a prototype, I will present mathematical studies of the transmissionenhancement in the subwavelength structures. Based upon the layer potential technique, asymptotic analysis, different types of enhancement mechanism are unveiled, which includes Fabry-Perot resonance, surface modes, and Fano resonance.

Thick-Restart Lanczos with Explicit External Deflation for Computing Many Egenpairs,

and its Communication-Avoiding Variant

Zhaojun Bai

University of California, Davis

Abstract: There are continual and compelling needs for computing many eigenpairs of very large Hermitian matrices in physical simulations and data analysis. Though the Lanczos method is effective for computing a few eigenvalues, it can be expensive for computing a large number of eigenvalues. To improve the performance of the Lanczos method, in this talk, we will present a combination of explicit external deflation (EED) with an s-step variant of thick-restart Lanczos (s-step TRLan). The s-step Lanczos method can achieve an order of s reduction in data movement while the EED enables to compute eigenpairs in batches along with a number of other advantages. In addition, we will discuss the backward stability analysis of the EED.

Eigenfunction Behaviors and Adaptive Finite Element Approximations of Nonlinear Eigenvalue Problems

Aihui Zhou

Chinese Academy of Sciences

Abstract: In this presentation, we talk about the eigenfunction behaviors and approximations of nonlinear eigenvalue problems in quantum physics. We first prove that the eigenfunction cannot be a polynomial in any open set, which is a refinement of the classic unique continuation property. Then we apply non-polynomial behaviors of eigenfunctions to prove that the adaptive finite element approximation is convergent even if the initial mesh is not fine enough. We finally remark that the adaptive finite element method has linear convergence rate and optimal complexity.

Finite element/holomorphic operator value function approach for nonlinear eigenvalue problems

Jiguang Sun

Michigan Technological University

Abstract: We propose a new approach combining the holomorphic operator value function and finite elements for some nonlinear eigenvalue problems. The eigenvalue problem is formulated as the eigenvalue problem of a holomorphic Fredholm operator function of index zero. Finite element methods are used for discretization. The convergence of eigenvalues/eigenvectors is proved using the abstract approximation theory for holomorphic operator functions. Then the spectral indicator method is extended to compute the eigenvalues. The proposed approach is employed to compute the transmission eigenvalue problem arising from the inverse scattering theory for nonhomogeneous media.

A C0 finite element method for the biharmonic problem with the Navier boundary

condition

Hengguang Li

Wayne State University

Abstract: We study the biharmonic equation with the Navier boundary condition in a polygonal domain. In particular, we propose a method that effectively decouples the 4th-order problem into a system of Poisson equations. Different from the usual mixed method that leads to two Poisson problems but only applies to convex domains, the proposed decomposition involves a third Poisson equation to confine the solution in the correct function space, and therefore can be used in both convex and non-convex domains. A C0 finite element algorithm is in turn proposed to solve the resulted system. In addition, we derive the optimal error estimates for the numerical solution on both quasi-uniform meshes and graded meshes. Numerical test results justify the theoretical findings.

Resonances in Nano-Holes and Extraordinary Optical Transmission

Junshan Lin

Auburn University

Abstract: In this talk, I will focus on resonant scattering for nano-hole structures, which leads to the so-called extraordinary optical transmission (EOT) phenomenon widely used in bio-sensing, near-field imaging, etc. Based upon the layer potential technique, asymptotic analysis and the homogenization theory, I will present rigorous mathematical analysis for a variety of resonance mechanisms in two-dimensional nano-structures. These include Fabry-Perot resonance, Fano resonance, spoof surface plasmon, etc.

2DEVP: theory and algorithms

Yangfeng Su

Fudan University

The 2D eigenvalue problem (2DEVP) is a class of the 2-parameter eigenvalue problems. 2DEVP seeks real scalars λ, μ and a corresponding vector x satisfying the following 2DEVP equations

$$\begin{cases} Ax = \lambda x + \mu C x, \\ x^H C x = 0, \\ x^H x = 1, \end{cases}$$

where A and C are Hermitian and C is inde_nite.

Let $\lambda_1(\mu) \geq \lambda_2(\mu) \geq \cdots \geq \lambda_n(\mu)$ be eigenvalues of $\mathrm{A} - \mu C$. Then 2DEVP equations are essentially the first order condition for $\lambda_i(\mu)$ to reach local extremum, and closely related to the eigenvalue optimization. The aims of this study are to reveal more mathematical properties, find more efficient algorithms, from the point of algebra, for eigenvalue optimization.

In this talk, we will briefly introduce the applications and fundamental theory of 2DEVP, including its variational characterization and number of solutions. We will introduce two numerical methods to solve it. The first one is the 2D Rayleigh Quotient Iteration(2DRQI) method, which enjoys fast local convergence rate and is very suitable for large scale problems. We also

propose a corrected Newton method. Though it is less convenient to use compared to 2DRQI, the corrected Newton method overcomes the non-isolation of solutions and non-differentiation in complex equation, which provides insights for overcoming these difficulties in general cases. Examples are given to demonstrate the superiority of our algorithms.

Unfitted Computation of edge modes in photonic graphene

Hailong Guo

University of Melbourne

Abstract: Photonic graphene, a photonic crystal with honeycomb structures, has been intensively studied in both theoretical and applied fields. Similar to graphene which admits Dirac Fermions and topological edge states, photonic graphene supports novel and subtle propagating modes (edge modes) of electromagnetic waves. These modes have wide applications in many optical systems. In this paper, we propose a new unfitted Nitsche's method of computing edge modes in photonic graphene with some defect. The unique feather of the methods is that they can arbitrary handle high contrast with geometric unfitted meshes. We establish the optimal convergence of methods. Numerical examples are presented to validate the theoretical results and to numerically verify the existence of the edge modes.

Finite Element Calculation of Photonic Band Structures for Frequency Dependent Materials

Wenqiang Xiao

Beijing Computational Science Research Center

Abstract： Band structure calculation of frequency dependent photonic crystals has important applications. The associated eigenvalue problem is nonlinear and the development of effective convergent numerical methods is challenging. In this talk, we formulate the band structure problem as the eigenvalue problem of a holomorphic Fredholm operator function of index zero. Lagrange finite elements are used to discretize the operators. The convergence of the eigenvalues is proved using the abstract approximation theory for holomorphic operator functions. Then a spectral indicator method is developed to practically compute the eigenvalues. Numerical examples are presented to validate the theory and show the effectiveness of the proposed method.

An efficient spectral-Galerkin approximation based on dimension reduction scheme for transmission eigenvalues in polar geometries

Jing An

Beijing Computational Science Research Center

Abstract: We put forward an efficient spectral-Galerkin approximation in view of dimension reduction scheme for transmission eigenvalue problem in polar geometries. Firstly, we turn the

original problem into an equivalent fourth order nonlinear eigenvalue problem. Then the fourth order nonlinear eigenvalue problem is transformed into a coupled fourth order linear eigenvalue system by introducing an auxiliary Poisson equation. Secondly, based on polar coordinate transformation, we further reduce the coupled fourth order linear eigenvalue system to a series of equivalent one-dimensional eigenvalue systems. Thirdly, we derive the essential polar condition and introduce the appropriate weighted Sobolev space according to the polar condition, and establish the weak form and the corresponding discrete form. In addition, by utilizing spectral theory of compact operators, we prove the error estimates of approximation eigenvalues and eigenvectors for each one-dimensional eigenvalue system. Finally, we provide ample numerical experiments, and the numerical results show the effectiveness of the algorithm and the correctness of the theoretical results.

A contour-integral based method for computing the number of eigenvalues inside a region

Guojian Yin

Chinese University of Hong Kong

Abstract: In many applications, the information about the number of eigenvalues inside a given region, rather than the eigenvalues, is required. In this talk, I will introduce a contour-integral based method for this purpose. Our method is motivated by two findings. There exist methods for estimating the number of eigenvalues inside a region in the complex plane, but our method is able to compute the number exactly. Our method has a good potential to be implemented on a high-performance parallel architecture. Numerical experiments will be reported to show the viability of our method.

Fast Algorithms for Maxwell's Equations for Three-Dimensional Photonic Crystals

Tiexiang Li

Southeast University

Abstract: In this work, we propose the Fast Algorithms for Maxwell's Equations (FAME) package for solving Maxwell's equations for modeling three-dimensional photonic crystals. FAME combines the null-space free method with fast Fourier transform (FFT)-based matrix-vector multiplications to solve the generalized eigenvalue problems (GEPs) arising from Yee's discretization. The GEPs are transformed into a null-space free standard eigenvalue problem with a Hermitian positive-definite coefficient matrix. The computation times for FFT-based matrix-vector multiplications with matrices of dimension 7 million are only 0.33 and 3.6 x 10^{-3} seconds using MATLAB with an Intel Xeon CPU and CUDA C++ programming with a single NVIDIA Tesla P100 GPU, respectively. We successfully use FAME on a single P100 GPU to solve a set of GEPs with matrices of dimension more than 19 million, in 127 to 191 seconds per problem. These results demonstrate the potential of our proposed package to enable large-scale numerical simulations for novel physical discoveries and engineering applications of photonic crystals.

A spectral projection method for transmission eigenvalues

Fang Zeng

Chongqing University

Abstract: We consider a nonlinear integral eigenvalue problem, which is a reformulation of the transmission eigenvalue problem arising in the inverse scattering theory. The boundary element method is employed for discretization, which leads to a generalized matrix eigenvalue problem. We propose a novel method based on the spectral projection. The method probes a given region on the complex plane using contour integrals and decides whether the region contains eigenvalue(s) or not. It is particularly suitable to test whether zero is an eigenvalue of the generalized eigenvalue problem, which in turn implies that the associated wavenumber is a transmission eigenvalue. Effffectiveness and effifffifficiency of the method are demonstrated by numerical examples.

Convergence analysis of two finite element methods for the modified Maxwell’s Stekloff eigenvalue problem

Bo Gong

Beijing University of Technology

Abstract: The modified Maxwell's Stekloff eigenvalue problem recently arises from the inverse electromagnetic scattering theory. In this talk, we shall investigate two finite element methods for this problem and derive their convergence analysis.