Summer School Abstracts

Boundary layers in kinetic-fluid coupling

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Many kinetic equations have their corresponding fluid limits. In the zero limit of the Knudsen number, one derives the Euler equations from the Boltzmann equation and the heat equation from the radiative transfer equation. There are good numerical solvers for both kinetic and fluid equations respectively, the computation becomes significantly harder when the two regimes co-exist. The transition from one regime to the other is termed the Knudsen layer. In this talk, we model the layer using a half-space equation, study the well-posedness, design a numerical solver, and utilize it to couple the two sets of equations that govern separate domains. Joint work with Weiran Sun and Jianfeng Lu.

Related papers:

Validity and Regularization of Classical Half-Space Equations, Journal of Statistical Physics, 166 (2017)

A numerical method for coupling the BGK model and Euler equations through the linearized Knudsen layer, Journal of Computational Physics, 398, 2019

Diffusion approximations and domain decomposition method of linear transport equations: Asymptotics and numerics, Journal of Computational Physics, 292, 2015

A convergent method for linear half-space kinetic equations, M2AN 51 (2017)

Half-space kinetic equations with general boundary conditions, Math. Comp. 86 (2017)

Low rank structure in the forward and inverse kinetic theory

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Kinetic theory is a body of theory from statistical mechanics. It is useful in describing the dynamics of a large number of particles, but its high dimensional structure makes the computation infeasible. In multi-scale regimes, however, kinetic equations can be compressed: The Boltzmann equation is asymptotically equivalent to the Euler equations, and the radiative transfer equation is asymptotically equivalent to the diffusion equation. In linear algebra, this phenomenon is equivalent to a system being of low rank.

I will discuss how the low rank structure forms, and how it affects the computation. In the forward regime, inserting the low-rank structure greatly advances the computation, but in the inverse regime, the system being of low rank typically makes the problems significantly harder. Joint work with Ke Chen, Shi Chen, Christian Klingenberg, Ru-Yu Lai, Jianfeng Lu, Gunther Uhlmann, and Stephen Wright.

Related papers:

Reconstruction of the Emission Coefficient in the Nonlinear Radiative Transfer Equation, SIAM J. Appl. Math. 81(1) (2021), 91-106.

Inverse Problems for the Stationary Transport Equation in the Diffusion Scaling, SIAM J. Appl. Math. 79(6) (2019), 2340-2358.

Random Sampling and Efficient Algorithms for Multiscale PDEs, SIAM J. Sci. Comput. 42(5) (2020), A2974-A3005.

Manifold learning and Nonlinear homogenization, arXiv: 2011.00568., 2020.

The review of the Discrete Ordinate method for particle transfer equation

Wen-Jun Sun

Institute of Applied Physics and Computational Mathematics

In this lecture, we first introduce the Discrete Ordinate method for the angular variable of particle transfer equation. Then is the review of the fast iteration method for the resulting discreted equation. And last the detail of unified gas kinetic scheme for radiation transfer equation is presented.

Moment methods for rarefied gases

Zhen-Ning Cai
National University of Singapore, Singapore

Moment method is one of the important tools for the derivation of macroscopic transport equations. In our lectures, we will review the history of the moment methods applied to the gas kinetic theory and brief the recent progress in this direction. The topics include Grad's moment method and its regularization, the method of maximum entropy, the quadrature-based moment methods, and methods with large numbers of moments.

Discontinuous Galerkin method for the high order derivative nonlinear equations

Yan Xu
University of Science and Technology of China

In this talk, we discuss local discontinuous Galerkin method for solving the nonlinear wave equations which contain nonlinear high order derivatives. The discretization results in an extremely local, element based discretization, which is beneficial for parallel computing and maintaining high order accuracy on unstructured meshes. In particular, the methods are well suited for hp-adaptation, which consists of local mesh refinement and/or the adjustment of the polynomial order in individual elements. The stability and the error estimates of the numerical methods will be discussed. Numerical simulation results for different types of solutions illustrate the accuracy and capability of the methods.

Properties of some DG methods for a kinetic transport equation

Feng-Yan Li

Rensselaer Polytechnic Institute

In this lecture, I will consider a linear kinetic transport model under a diffusive scaling, and examine some theoretical properties (e.g. stability, asymptotic property etc) of several numerical methods that are based on discontinuous Galerkin (DG) methods in space, with temporal discretizations being either fully implicit or implicit-explicit.

Accurate and efficient Monte Carlo methods for radiative transfer equations

Yi Shi

Institute of Applied Physics and Computational Mathematics

The thermal radiation transport (TRT) equations, also known as radiative transfer, describe the dynamics of photon transport and its collision with the background material. The system of TRT equations has wide applications in the field of astrophysics, inertial confinement fusion (ICF), plasma physics and so on. The implicit Monte Carlo (IMC) method is a classical stochastic method for solving the TRT equations, and its solution remains the standard of high-fidelity radiative transfer simulations. In this talk, we will first present an overview of the radiation transport equations. Next, we introduce the IMC equations and describe the equations with the Monte Carlo interpretation. Then we will discuss the numerical limitations of the IMC method for its existence of teleportation error in optically thick regions and the violation of maximum principle for large time steps. We present some numerical algorithms to improve the spatial accuracy and temporal stability of the IMC method. Lastly, we will discuss the newly developed unified gas kinetic particle (UGKP) method. The UGKP method utilizes a system of macroscopic equations to accelerate the microscopic transport equation, and it’s much more efficient than the IMC method in optically thick regions. We prove that the UGKP method has the asymptotic preserving property in both the free transport and diffusion regimes.

A brief introduction to the Boltzmann equation and its numerical methods

Jing-Wei Hu
Purdue University

In this short course, I will give a brief introduction to the Boltzmann equation including its derivation and basic properties. Then I will introduce the deterministic numerical methods, in particular, the Fourier spectral method, for solving the Boltzmann equation. Both the fast implementation and stability/convergence of the method will be discussed.