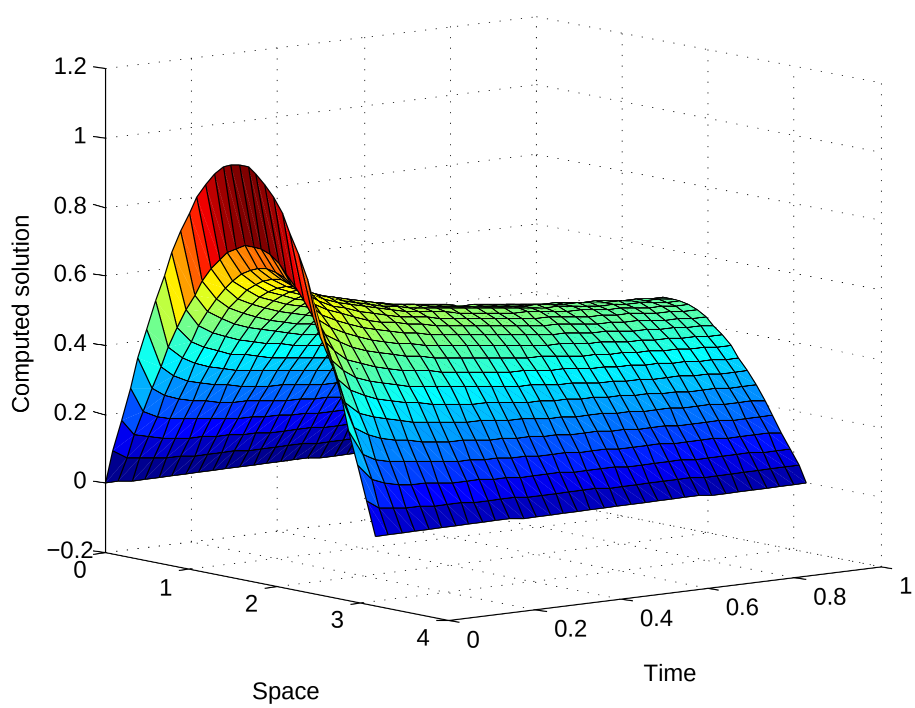
Book of Abstracts

8th Annual Conference

Numerical Methods for Fractional-Derivative Problems

Beijing Computational Science Research Center

08–11 July 2024



8th CSRC Conference

Numerical Methods for Fractional-Derivative Problems

8-11 July 2024

Each time slot: $\boxed{\textit{40 minutes for the talk}}$ *+ 5 minutes for questions and change of speakers*

All lectures are in Conference Room 1 (after entering CSRC, walk to the right)

Registration takes place outside Conference Room 1 on Monday 8 July 14:00–17:00 and Tuesday 9 July 08:00–08:30.

Chairpersons of sessions: Please check the list at the end of this Program to see if you are requested to act as Chairperson for any session. The conference organisers hope that the people on this list are willing to accept this responsibility. If you are unable to do so, please let the organisers know as soon as possible.

Tuesday 9 July	
08:30–08:35	*Opening of Conference*
08:35–09:20	**Lei Li**, Resolvent kernels and monotonicity-preserving discretizations of Volterra equations
09:20–10:05	**Hui Liang**, Analysis of direct piecewise polynomial collocation methods for the Bagley-Torvik equation
10:05–10:30	*Tea/Coffee Break*
10:30–11:15	**Wanrong Cao**, High-order numerical methods for the fractional Langevin equation via Wong-Zakai approximation and its extension
11:15–12:00	**Zhi Zhou**, Inverse problems for subdiffusion from observation at an unknown terminal time
12:00–13:45	*Lunch (CSRC Canteen, in basement)*
13:50–14:00	*Group photo*
14:00–14:45	**Seakwen Vong**, Numerical study on multi-singularity problems arising from time delay fractional equations
14:45–15:30	**Weiping Bu**, Finite element method for a generalized constant delay diffusion equation
15:30–16:00	*Tea/Coffee Break*
16:00–16:45	**Meng Li**, Finite element methods for nonlinear time-fractional parabolic equations with constant/distributed delay
16:45–17:30	**Hu Chen**, Grünwald-Letnikov scheme for a multi-term time fractional reaction-subdiffusion equation
17:45	*Dinner (CSRC Canteen, in basement)*

Wednesday 10 July	
08:30–09:15	**Tao Zhou**, Deep adaptive density approximation for classic/nonlocal Fokker-Plank type equations
09:15–10:00	**Dongfang Li**, An energy-stable and divergence-free variable-step L1 scheme for time-fractional Navier-Stokes equations
10:00–10:30	*Tea/Coffee Break*
10:30–11:15	**Xuan Zhao**, Energy dissipation and evolutions of the nonlocal Cahn-Hilliard model and space fractional variants using efficient variable-step BDF2 method
11:15–12:00	**Dongdong Hu**, Linearly implicit schemes preserve the maximum bound principle and energy dissipation for the time-fractional Allen-Cahn equation
12:00–14:00	*Lunch (CSRC Canteen, in basement)*
14:00–14:45	**Hongfei Fu**, A fast fractional block-centered finite difference method for two-sided space-fractional diffusion equations on general nonuniform grids
14:45–15:30	**Shuowan Wu**, Monotone discretization of integral fractional Laplacian on bounded Lipschitz domains: applications to the fractional obstacle problem
15:30–16:00	*Tea/Coffee Break*
16:00–16:45	**Minghua Chen**, Error analysis of a collocation method on graded meshes for a fractional Laplacian problem
16:45–17:30	**Xiangcheng Zheng**, Two methods addressing variable-exponent fractional initial and boundary value problems and Abel integral equation
17:45	*Dinner (CSRC Canteen, in basement)*

Thursday 11 July	
08:30–09:15	**Natalia Kopteva**, Error analysis for higher-order methods for subdiffusion equations on quasi-graded meshes
09:15–10:00	**Chaoyu Quan**, H^1-norm stability and convergence of an L2-type method on nonuniform meshes for subdiffusion equation
10:00–10:30	*Tea/Coffee Break*
10:30–11:15	**Fanhai Zeng**, Fast time-stepping discontinuous Galerkin method for the subdiffusion equation
11:15–12:00	**Dongling Wang**, Numerical Mittag-Leffler stability of initial steps correction schemes for sub-diffusion equations
12:00–12:10	*Closing of Conference*
12:10–13:00	*Lunch (CSRC Canteen, in basement)*

Chairpersons of sessions	
Tuesday 08:35–10:05	Dongfang Li
Tuesday 10:30–12:00	Tao Zhou
Tuesday 14:00–15:30	Meng Li
Tuesday 16:00–17:30	Seakwen Vong
Wednesday 08:30–10:00	Xuan Zhao
Wednesday 10:30–12:00	Chaoyu Quan
Wednesday 14:00–15:30	Minghua Chen
Wednesday 16:00–17:30	Hongfei Fu
Thursday 08:30–10:00	Fanhai Zeng
Thursday 10:30–12:00	Zhi Zhou

Finite element method for a generalized constant delay diffusion equation

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Finite element method is considered to solve a generalized constant delay diffusion equation. The regularity of the solution of the considered model is investigated, which is discovered that the solution has non-uniform multi-singularity in time. To overcome the multi-singularity, a symmetrical graded mesh is used to devise the fully discrete finite element scheme for the considered problem based on L1 formula of the Caputo fractional derivative and fractional trapezoidal formula of the Riemann-Liouville fractional integral. Then we investigate the unconditional stability of this scheme. Next, the local truncation errors of the L1 formula and the fractional trapezoidal formula are analyzed in detail under the multi-singularity of the solution and the symmetrical graded mesh. Using these error results, we obtain the convergence of the proposed numerical scheme. Finally, some numerical tests are provided to verify the obtained theoretical results.

KEY WORDS: Fractional constant delay diffusion equation, multi-singularity, finite element method, stability and convergence.

High-order numerical methods for the fractional Langevin equation via Wong-Zakai approximation and its extension

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A full-discrete scheme is presented for a Langevin equation involving Caputo fractional derivative and additive white noise. Based on a spectral truncation of white noise, the fractional Langevin equation is converted to an approximate equation with random parameters and a finite difference scheme is constructed. Consistency of the approximate equation as well as error estimate of the finite difference scheme are obtained. It is proved that when spectral truncation level and the step size are inversely proportional, the convergence order of the finite difference scheme is 1.5 in mean-square sense and independent of the value of fractional derivative order. Numerical examples verify the theoretical analysis. Moreover, to fulfill long-time simulation, a scheme based on piecewise spectral approximation of white noise is further developed.

KEY WORDS: stochastic differential equations, Caputo fractional derivative, Wong-Zakai approximation, finite difference method, Laplace and inverse Laplace transform

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2. Chengqiang Xu, Wanrong Cao*, Numercial analysis for stochastic time-fractional Allen-Cahn equation driven by fractional additive Gaussian noise, submitted.

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Grünwald-Letnikov scheme for a multi-term time fractional reaction-subdiffusion equation

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In this work, we establish the pointwise-in-time error estimate of a Grünwald-Letnikov scheme for a multi-term time fractional reaction-subdiffusion problem with initial singularity, where Legendre spectral Galerkin method is used for spatial discretization. The theoretical results show that the temporal accuracy is first order if t is away from 0, while, globally it is $O(\tau^{\alpha_1})$, where α_1 is the highest order of the multi-term temporal Caputo fractional derivatives. Numerical results are presented to show the sharpness of the error estimate.

KEY WORDS: Grünwald-Letnikov scheme, pointwise-in-time error estimate, reaction-subdiffusion, time fractional

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¶This work was supported in part by Natural Science Foundation of Shandong Province under Grant ZR2023MA077, the National Natural Science Foundation of China under Grant 11801026, and Fundamental Research Funds for the Central Universities, China (No. 202264006).

Error analysis of a collocation method on graded meshes for a fractional laplacian problem

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The numerical solution of a 1D fractional Laplacian boundary value problem is studied. Although the fractional Laplacian is one of the most important and prominent nonlocal operators, its numerical analysis is challenging, partly because the problem's solution has in general a weak singularity at the boundary of the domain. To solve the problem numerically, we use piecewise linear collocation on a mesh that is graded to handle the boundary singularity. A rigorous analysis yields a bound on the maximum nodal error which shows how the order of convergence of the method depends on the grading of the mesh; hence, one can determine the optimal mesh grading. Numerical results are presented that confirm the sharpness of the error analysis.

KEY WORDS: Fractional Laplacian, collocation method, graded meshes, error analysis

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¶The work of the first author was supported by the Science Fund for Distinguished Young Scholars of Gansu Province under Grant No. 23JRRA1020.

A fast fractional block-centered finite difference method for two-sided space-fractional diffusion equations on general nonuniform grids

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In this work, a two-sided variable-coefficient space-fractional diffusion equation with fractional Neumann boundary condition is considered. To conquer the weak singularity caused by the nonlocal space-fractional differential operators, by introducing an auxiliary fractional flux variable and using piecewise linear interpolations, a fractional block-centered finite difference (BCFD) method on general nonuniform grids is proposed. However, like other numerical methods, the proposed method still produces linear algebraic systems with unstructured dense coefficient matrices under the general nonuniform grids. Consequently, traditional direct solvers such as Gaussian elimination method shall require $\mathcal{O}(M^2)$ memory and $\mathcal{O}(M^3)$ computational work per time level, where M is the number of spatial unknowns in the numerical discretization. To address this issue, we combine the well-known sum-of-exponentials (SOE) approximation technique with the fractional BCFD method to propose a fast version fractional BCFD algorithm. Based upon the Krylov subspace iterative methods, fast matrix-vector multiplications of the resulting coefficient matrices with any vector are developed, in which they can be implemented in only $\mathcal{O}(MN_{exp})$ operations per iteration, where $N_{exp} \ll M$ is the number of exponentials in the SOE approximation. Moreover, the coefficient matrices do not necessarily need to be generated explicitly, while they can be stored in $\mathcal{O}(MN_{exp})$ memory by only storing some coefficient vectors. Numerical experiments are provided to demonstrate the efficiency and accuracy of the method.

KEY WORDS: Space-fractional diffusion equations, fractional block-centered finite difference method, fast matrix-vector multiplication, nonuniform grids.

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Linearly implicit schemes preserve the maximum bound principle and energy dissipation for the time-fractional Allen-Cahn equation

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This talk presents two highly efficient numerical schemes for the time-fractional Allen-Cahn equation that preserve the maximum bound principle and energy dissipation in discrete settings. To this end, we utilize a generalized auxiliary variable approach proposed in a recent work (Ju et al., 2022) to reformulate the governing equation into an equivalent system that follows a modified energy functional and the maximum bound principle at each continuous level. By combining the L1-type formula of the Riemann-Liouville fractional derivative with the Crank-Nicolson method, we construct two novel linearly implicit schemes by introducing the first- and second-order stabilized terms, respectively. These schemes are proved to be energy stable and maximum bound principle preserving on arbitrary nonuniform time meshes with the help of the discrete orthogonal convolution technique. In addition, we obtain the unique solvability of the proposed schemes without any time-space step ratio. Finally, we report extensive numerical results to verify the correctness of the theoretical analyses and the performance of the proposed schemes in long-time simulations.

KEY WORDS: Time-fractional Allen-Cahn equation, energy-stable scheme, maximum bound principle, linearly implicit scheme, orthogonal convolution kernel, adaptive time-stepping

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Error analysis for higher-order methods for subdiffusion equations on quasi-graded meshes

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An initial-boundary value problem with a Caputo time derivative of fractional order $\alpha \in (0,1)$ is considered, solutions of which typically exhibit a singular behaviour at an initial time. I will start with a review of [1] and [2], where we give a simple and general numerical-stability analysis using barrier functions, which yields sharp pointwise-in-time error bounds on quasi-graded temporal meshes with arbitrary degree of grading. This approach was initially employed in the error analysis of the L1 method. This methodology is also generalized for semilinear fractional parabolic equations [4].

The main focus of the talk will be on higher-order discretizations, such as the Alikhanov L2-1_σ scheme, also considered in [2], and an L2-type discretization of order $3-\alpha$ in time [3]. Some recent results for the latter will also be presented. The theoretical findings are illustrated by numerical experiments.

KEY WORDS: fractional-order parabolic equation, quasi-graded temporal grid, L1 scheme, Alikhanov scheme, L2 scheme, semilinear subdiffusion equaitons

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4. N. Kopteva, Error analysis for time-fractional semilinear parabolic equations using upper and lower solutions, SIAM J. Numer. Anal., 58 (2020), 2212–2234.

An energy-stable and divergence-free variable-step L1 scheme for time-fractional Navier-Stokes equations

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We propose a structure-preserving scheme and its error analysis for time-fractional Navier-Stokes equations (TFNSEs) with periodic boundary conditions. The equations are first rewritten as an equivalent system by eliminating the pressure explicitly. Then, the spatial and temporal discretization are done by the Fourier spectral method and variable-step L1 scheme, respectively. It is proved that the fully-discrete scheme is energy-stable and divergence-free. The energy is an asymptotically compatible one since it recovers the classical energy when $\alpha \to 1$. Moreover, optimal error estimates are presented very technically by the obtained boundedness of the numerical solutions and some Sobolev inequalities. To our knowledge, they are the first results of the construction and analysis of structure-preserving schemes for TFNSEs. Several interesting numerical examples are given to confirm the theoretical results at last.

KEY WORDS: Time-fractional Navier-Stokes equation, variable-step L1 scheme, energy stability, adaptive time-stepping strategy, error estimation

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1. R. Gao, D. Li, Y. Li, Y. Yin, An energy-stable and divergence-free variable-step L1 scheme for time-fractional Navier-Stokes equations, Phys D. (to appear).

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Resolvent kernels and monotonicity-preserving discretizations of Volterra equations

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We investigate the discretizations of Volterra integral equations on nonuniform meshes using the resolvent kernels. We first consider equations with completely positive kernels and give descriptions to such kernels for nonuniform meshes. Second, we investigate the monotonicity properties of the solutions for equations in R with nonincreasing and completely positive kernels, which we call complementary monotone kernels. Through the complementary kernels and the resolvents, we show the monotonicity preserving properties of the numerical solutions. We apply our theory to some discretizations of time fractional differential equations.

KEY WORDS: Resolvent,Convolution,Complete positivity,Nonuniform mesh,Fractional differential equations

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Finite element methods for nonlinear time-fractional parabolic equations with constant/distributed delay

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In this work, we propose efficient finite element methods for the nonlinear time-fractional parabolic equations with constant/distributed delay. For the constructed numerical schemes, we mainly focus on the unconditional convergence and superconvergence analysis of the numerical schemes without any time-space ratio restrictions. Numerical tests for several biological models, including the fractional single-species population model with delay, the fractional diffusive Nicholson's blowflies equation with delay and the fractional diffusive Mackey-Glass equation with delay, are conducted to confirm the theoretical results.

KEY WORDS: Nonlinear time-fractional parabolic equations with distributed delay, nonuniform meshes, error splitting, convergence and superconvergence

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Analysis of direct piecewise polynomial collocation methods for the Bagley-Torvik equation

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The piecewise polynomial collocation method does not always work for Caputo fractional differential equations (FDEs), since it is related to the well-known Conjecture 6.3.5 in Brunner's 2004 monograph on the convergence of the collocation solution for weakly singular Volterra integral equations (VIEs) of the first kind, and this is the reason why in the existing literature, the collocation method is not used directly to solve FDEs, but rather indirectly to solve the reformulated VIEs. The Bagley-Torvik (BT) equation is a typical representative of a class of FDEs, whose highest order derivative of the unknown function is an integer, and a Caputo derivative is also involved, and the characteristic with dominant integer order derivative allows us to use collocation methods directly. The existence, uniqueness and regularity of the exact solution for this problemare given by virtue of the theory of VIEs, but the piecewise polynomial collocation method is used directly to solve the BT equation, and the global convergence is derived on graded meshes and the pointwise error estimate is obtained on uniform meshes. Moreover, the global superconvergence of the collocation solution is also obtained without any postprocessing techniques. Unlike the indirect reformulated numerical methods, one has to resort to the iterated numerical solution to improve the numerical accuracy. Numerical examples illustrate the theoretical results, and it also shows that our analysis for the BT equation can be extended to more general integer order derivative dominant FDEs, even for time fractional partial differential equation with this characteristic.

KEY WORDS: Bagley-Torvik equation, collocation method, convergence, superconvergence

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1. Lu Wang and Hui Liang, Analysis of direct piecewise polynomial collocation methods for the Bagley-Torvik equation, BIT Numerical Mathematics, 2024. (submit).

¶The work was supported by the National Natural Science Foundation of China (Grant No. 12171122), Guangdong Provincial Natural Science Foundation of China (2023A1515010818) and Shenzhen Science and Technology Program (RCJC20210609103755110).

H^1-norm stability and convergence of an L2-type method on nonuniform meshes for subdiffusion equation

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In this work the H^1-stability of an L2 method on general nonuniform meshes is established for the subdiffusion equation. Under some mild constraints on the time step ratio ρ_k, for example $0.4573328 \leq \rho_k \leq 3.5615528$ for all $k \geq 2$, a crucial bilinear form associated with the L2 fractional-derivative operator is proved to be positive semidefinite. As a consequence, the long time H^1-stability of L2 schemes can be derived for the subdiffusion equation. In the case of standard graded mesh, such positive semidefiniteness and H^1-stability hold when the grading parameter $1 < r \leq 3.2016538$. Based on the positive definiteness results, the error analysis in H^1-norm for general nonuniform meshes is provided and particularly the convergence of order $(5-\alpha)/2$ in H^1-norm is proved for modified graded meshes when $r > 5/\alpha - 1$. To the best of our knowledge, this is the first work on the H^1-norm stability and convergence of L2 method on general nonuniform meshes for subdiffusion equation.

KEY WORDS: L2-type method, subdiffusion equation, graded mesh, positive semidefiniteness, H^1-norm stability and convergence

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Numerical study on multi-singularity problems arising from time delay fractional equations

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In this talk, we study delay fractional equations. We show that the regularity of the solution at s_+ is better than that at 0_+, where s is a constant time delay. Improved regularity of the solution is obtained by the decomposition technique and a fitted L1 numerical scheme is designed for it. We then construct two corrected L-type schemes, of which optimal convergence order reaches $2-\alpha$ and 2, respectively, where $\alpha \in (0,1)$ is the order of the Caputo derivative. Significantly, the correction terms share the same forms as the discrete convolution structure for the derivative, which implies that the computation and analysis of these two parts can be integrated together.

KEY WORDS: Delay fractional equations, multi-singularity problem

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Numerical Mittag-Leffler stability of initial steps correction schemes for sub-diffusion equations

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Compared with the standard diffusion model, the solution of the sub-diffusion equation possesses two distinctive characteristics. Firstly, the solution exhibits a weak singularity at the initial moment, leading to a reduction in the convergence order of most conventional numerical methods on uniform time meshes. An effective method to restore the high-order convergence rate of numerical methods is the initial steps correction technique. Secondly, the solution exhibits an algebraic decay rate over a long time interval as opposed to the exponential decay rate observed in the standard diffusion equation, a phenomenon commonly known as Mittag-Leffler (ML) stability. Building upon the basic idea of structure-preserving algorithms, it is advantageous for the numerical solution to simultaneously preserve the key qualitative features of the original sub-diffusion equation. We derive criteria for various initial steps correction methods to preserve long-term ML stability, including fractional BDFk, L1 method, Crank-Nicolson, etc. Our main method is the singularity analysis of generating function. This result enables numerical methods to achieve a high order convergence rate while also preserving long-term ML stability. Numerical experiments are provided to validate our theoretical analysis.

KEY WORDS: Sub-diffusion equations, weak singularity kernel, initial steps correction, Mittag-Leffler stability, generating function, singularity analysis.

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Monotone discretization of integral fractional laplacian on bounded lipschitz domains: applications to the fractional obstacle problem

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We propose a monotone discretization method for the integral fractional Laplacian on bounded Lipschitz domains with homogeneous Dirichlet boundary conditions, specifically designed for solving fractional obstacle problems. Operating on unstructured grids in arbitrary dimensions, the method offers flexibility in approximating singular integrals over a domain that depends not only on the local grid size but also on the distance to the boundary, where the Hölder coefficient of the solution deteriorates. Using a discrete barrier function reflecting the distance to the boundary, we demonstrate optimal pointwise convergence rates in terms of the Hölder regularity of the data on quasi-uniform and graded grids.

Applying this monotone discretization to the (nonlinear) fractional obstacle problems, we establish the uniform boundedness, existence, and uniqueness of numerical solutions. Monotonicity naturally implies the convergence of the policy iteration. Subsequently, based on the nature of this problem, an improved policy iteration tailored to solution regularity is devised, exhibiting superior performance through adaptive discretization across diverse regions. Several numerical examples are provided to illustrate the sharpness of the theoretical results and the efficacy of the proposed method.

KEY WORDS: Monotone discretization, bounded Lipschitz domains, unstructured grids, pointwise error estimate, fractional obstacle problem

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Fast time-stepping discontinuous Galerkin method for the subdiffusion equation

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The nonlocality of the fractional operator causes numerical difficulties for long time computation of the time-fractional evolution equations. This paper develops a high-order fast time-stepping discontinuous Galerkin finite element method for the time-fractional diffusion equations, which saves storage and computational time. The optimal error estimate $O(N^{-p-1} + h^{m+1} + \varepsilon N^{r\alpha})$ of the current time-stepping discontinuous Galerkin method is rigorous proved, where N denotes the number of time intervals, p is the degree of polynomial approximation on each time subinterval, h is the maximum space step, $r \geq 1$, m is the order of finite element space, and $\varepsilon > 0$ can be arbitrarily small. Numerical simulations verify the theoretical analysis.

KEY WORDS: Fast time-stepping discontinuous Galerkin method, optimal convergence, subdiffusion, nonlocality.

Energy dissipation and evolutions of the nonlocal Cahn-Hilliard model and space fractional variants using efficient variable-step BDF2 method

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In this work, an energy stable BDF2 scheme with general nonuniform time steps is developed for the nonlocal Cahn-Hilliard model to capture the multi-scale behavior of evolution efficiently and accurately [1]. The fully discrete numerical scheme is demonstrated to inherit main physical properties of the continuous model, namely, mass conservation and energy dissipation. Recently, various variants of the Cahn-Hilliard model with nonlocal operators and fractional Laplacian have attracted great attention in terms of theoretical analysis and numerical approximation, while there is less emphasis on the comparisons among the different variant models. By comparing the impact of various parameters and coarsening behaviors, numerical results suggest that the nonlocal Cahn-Hilliard model bears a strong resemblance to the space fractional Cahn-Hilliard model derived from H^{-1} gradient flow of the nonlocal energy. Nevertheless, the order of the fractional Laplacian appears to affect only the rate of energy decay, without altering the profile of the solution for the fractional model relating to the $H^{-\beta}(0 < \beta < 1)$ gradient flow of the classical Ginzburg-Landau energy functional in one dimensional case.

KEY WORDS: Nonlocal Cahn-Hilliard equation, variable-step BDF2, mass conservation, energy stable, adaptive time-stepping

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Two methods addressing variable-exponent fractional initial and boundary value problems and Abel integral equation

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Variable-exponent fractional models attract increasing attentions in various applications, while the rigorous analysis is far from well developed. For instance, it is commented in [Kian et al. 2024] that the analysis of variable-exponent subdiffusion remains an open problem. We provide general tools to address these models. Specifically, we first develop a convolution method to study the well-posedness, regularity, an inverse problem and numerical approximation for the sundiffusion of variable exponent. For models such as the variable-exponent two-sided space-fractional boundary value problem (including the variable-exponent fractional Laplacian equation as a special case) and the distributed variable-exponent model, for which the convolution method does not apply, we develop a perturbation method to prove their well-posedness. The relation between the convolution method and the perturbation method is discussed, and we further apply the latter to prove the well-posedness of the variable-exponent Abel integral equation and discuss the constraint on the data under different initial values of variable exponent.

KEY WORDS: Fractional calculus, variable exponent, analysis, numerical method

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Deep adaptive density approximation for classic/nonlocal Fokker-Plank type equations

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We propose adaptive deep learning method based on normalizing flow for classic/nonlocal Fokker-Planck equations. The solution of such equation is a probability density function. Traditional mesh-based methods may across difficulties since the dimension of spatial variable can be very high. To this end, we represent the solution by a flow-based generative model (e.g. KRnet) which constructs a mapping from a simple distribution to the target distribution (i.e., the unknow solution). An adaptive procedure for choosing the training set is presented. Meanwhile, either Monte Carlo sampling or an auxiliary density model, Gaussian radial basis functions which have analytical fractional Laplacian, is applied to approximate the fractional Laplacian. Numerical examples are presented to show the effectiveness of the proposed approach. Finally, we design bounded KRnet and show applications for solving Keller-Segel equations and kinetic Fokker-Planck equations.

KEY WORDS: Normalizing flow, Fokker-Planck equations, Density approximation, Adaptive sampling

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Inverse problems for subdiffusion from observation at an unknown terminal time

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Time-fractional subdiffusion equations represent an important class of mathematical models with a broad range of applications. The related inverse problems of recovering space-dependent parameters, e.g., initial condition, space dependent source or potential coefficient, from the terminal observation have been extensively studied in recent years. However, all existing studies have assumed that the terminal time at which one takes the observation is exactly known. In this talk, we present uniqueness and stability results for three canonical inverse problems, e.g., backward problem, inverse source and inverse potential problems, from the terminal observation at an unknown time. The subdiffusive nature of the problem indicates that one can simultaneously determine the terminal time and space-dependent parameter.

KEY WORDS: backward subdiffusion, inverse source problem, inverse potential problem, subdiffusion, unknown terminal time, uniqueness, stability

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